Higher Order Corrections to Radiation Reaction

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Abstract

The main objective of our project was to calculate higher-order corrections to the Abraham-Lorentz (AL) force. In a previous paper by D'Andrea et al. [1], they showed that, classically, one can systematically expand the AL force under the right conditions to obtain an effective AL equation. We want to demonstrate that we can obtain these corrections to the AL force using the formalism of Kosower et al. [2]. Currently, we do not have any conclusive results. Our project is still ongoing, and we have several ideas that will be detailed in the **Discussion** for future work.

Contents

1 Introduction

When charged particles accelerate, they must lose energy and momentum through the emission of electromagnetic waves. This process causes the particles to experience an additional force to the Lorentz Force, a phenomena known as radiation reaction. There are many subtleties in this topic, which are discussed in detail by D'Andrea et al. [1]. However, we will only be discussing the main issues of radiation reaction and how our project is related.

Classically, the effects of radiation reaction on charged particles are characterized by the AL force, which is proportional to the time derivative of the acceleration. When incorporating the AL force, solving the equations of motion for charged, point particles, yields *runaway* solutions. These solutions predict that the particle's velocity should increase to near the speed of light on a time scale r_c/c, where r_c is the classical radius of our charged particle and c is the speed of light. r_c is given by

$$r_c = \frac{q^2}{4\pi m}$$

, where q is the charge of our particle and m is its mass. For an electron, $r_c \sim 10^{-15}$ m, which implies that $r_c/c \sim 10^{-23}$ s. These solutions are in great contrast to our everyday observations.

To better understand this issue, D'Andrea et al. [1] modeled the charge particle as a spherical shell with some radius r_0 and looked at the limit as $r_0 \to 0$. Using conservation of energy and momentum, they derived a simplified equation of Newton's 2^{nd} Law involving the Lorentz Force and the AL Force, given below (taken from [1])

$$\left(m_0 + \frac{q^2}{6\pi r_0}\right)\vec{a} = q\vec{E}_{ext} + \frac{q^2}{6\pi}\dot{\vec{a}}\ . \tag{1}$$

The mass term, m, includes $\frac{q^2}{6\pi r_0}$ due to the fact that our charged particle has some size. m_0 is the 'bare' mass of our particle, which will be important later. On the right-hand side, the first term represents the Lorentz Force due to an external electric field, $\vec{E}_{ext}$, and the second term is the AL force. When looking at the energy and momentum of our particle, the only mass term that contributes is m_0, which was shown by D'Andrea et al. [1]. Further, in Eq. (1), if we look at the limit as $r_0 \to 0$, we see that $q^2/6\pi r_0 \to \infty$. Note, in addition to m being the sum of the two terms, it is a physical quantity that we can measure, thus making it a constant. Therefore, $m_0 \to -\infty$.

Effectively, this implies that the particle can lose energy and momentum but still accelerate to near the speed of light, since gaining speed allows its energy and momentum to become more negative. Clearly, we cannot use the point-particle assumption for our classical equations of motion, so D'Andrea et al. [1] assumed that $r_0 \gg r_c$. To still incorporate the point-particle description, they also assumed that the physical size of the charged particle was smaller than the length and time scales of the problem. They define L and T to be the characteristic length and time scales of their problem, respectively, to get the following condition on r_0.

$$r_c \ll r_0 \ll L, T \tag{2}$$

With relation (2), they were able to expand to higher orders in the radiation reaction force, which includes higher order acceleration terms with model-dependent coefficients, C_i.

$$\vec{a} = \frac{\vec{F}_{ext}}{m} + \frac{q^2}{6\pi m}\dot{\vec{a}} + C_1\frac{q^2}{6\pi m}r_0\ddot{\vec{a}} + C_2\frac{q^2}{6\pi m}r_0 a^2\vec{a} + C_3\frac{q^2}{6\pi m}r_0^2\dddot{\vec{a}} + ... \tag{3}$$

In the classical derivation, the C_is depend on the shape of the particle, hence why they are model-dependent. However, in our project, we make the argument that, due to quantum mechanics, our charged particle should have some inherent size given by the spread of its wavefunction. Therefore, we want to use quantum field theory (QFT) to derive these higher order terms.

2 Technical Background

To derive these terms, we looked at the formalism given by Kosower et al. (KMOC) [2]. Their paper formulated how to calculate the expected impulse from scattering amplitudes and properly take the classical limit of the quantum calculations. They were focused on calculating observables in gravitational waves coming from black hole and neutron star mergers. However, they also emphasized that their method can be applied to electrodynamics. To use their formalism, I will first describe which variables we are matching from our research to theirs and the methods necessary to do the calculations. From here on, we will be using relativistic natural units where $c = 1$, unless otherwise specified.

2.1 Matching Variables

KMOC defines l_w as the intrinsic spread of the wavefunction of the quantum particle, which we think of as our r_0 in the classical derivation. They also define a dimensionless quantity

$$\xi \equiv \left(\frac{l_c}{l_w}\right)^2 \ ,$$

where $l_c = \hbar/m$ is the reduced Compton wavelength, and make the argument that $l_w \gg l_c$ in the classical limit. Therefore, $\xi \to 0$ in the classical limit and is interpreted as a quantum correction to our observable of interest along with $\hbar$. However, we should check that this agrees with the left-hand side of relation (2). When we rearrange r_c, we obtain

$$r_c = \frac{\hbar e^2}{4\pi\hbar m} = \alpha l_c \tag{4}$$

, where α is the fine structure constant; thus, $l_w \gg r_c$.

2.2 Methodology for Calculations

We are interested in looking at the scattering of a single particle, since D'Andrea et al. [1] looked at the equations of motion of a single charged particle. Therefore, using the KMOC formalism [2], we define our incoming state as

$$|\psi\rangle = \int (dp)\psi(p)\,|p\rangle \ , \tag{5}$$

where (dp) is our Lorentz invariant measure

$$(dp) = \frac{d^4p}{(2\pi)^4}(2\pi)\delta(p^2 - m^2)\Theta(p^0) \ ,$$

$\psi(p)$ is the wavefunction of our particle, and $|p\rangle$ is the momentum eigenstate of our particle from our Lagrangian (which we'll identify as our model later on). Additionally, our observable of interest is the change in the expected value of the four momentum, $\langle \Delta p^\mu \rangle$. KMOC makes the argument that the expected value should return to the classical impulse as we take the classical limit, similar to our understanding of how quantum mechanics relates to classical mechanics. Classically (without relativistic effects), we know that $\vec{F} = \dot{\vec{p}}$, so calculating the impulse is relatively simple in this regime and can be matched on with the impulse from the quantum calculation. Using the fact that $|\psi\rangle_{out} = S\,|\psi\rangle_{in}$, where S is the time-evolution operator in QFT, we can write

$$\begin{aligned}
\langle \Delta p^\mu \rangle &= \langle p^\mu_{out} \rangle - \langle p^\mu_{in} \rangle \\
&= {}_{out}\langle \psi|\,\mathbb{P}\,|\psi\rangle_{out} - {}_{in}\langle \psi|\,\mathbb{P}\,|\psi\rangle_{in} \\
&= {}_{in}\langle \psi|\,S^\dagger \mathbb{P} S\,|\psi\rangle_{in} - {}_{in}\langle \psi|\,\mathbb{P}\,|\psi\rangle_{in} \\
&= \langle \psi|\,S^\dagger \mathbb{P} S\,|\psi\rangle - \langle \psi|\,\mathbb{P}\,|\psi\rangle \ ,
\end{aligned} \tag{6}$$

where $\mathbb{P}$ is our momentum operator and $|\psi\rangle$ is understood to be the incoming state, $|\psi\rangle_{in}$, in the last line. S can be written as $S = 1 + iT$, where 1 here is the unit matrix and T is the T-matrix. S is an operator, so it only acts on eigenstates in our theory. This allows us to isolate the interaction, which is related to our scattering amplitude. We denote the scattering amplitude, $\mathcal{M}$, of a specified initial state, $|i\rangle$, scattering to a specified final state, $|f\rangle$, as follows,

$$\begin{aligned}
\langle f|\,S\,|i\rangle &= \langle f|\,iT\,|i\rangle \\
&= i\mathcal{M}(i \to f) \ .
\end{aligned} \tag{7}$$

In most cases, it is impossible to completely solve for the interaction between fields. Therefore, we characterize our interaction with a coupling constant that allows us to use perturbation theory. Further, we solve $\mathcal{M}(i \to f)$ up to a certain order in our coupling constant. This will be discussed more when we touch on the Feynman Diagrams we have calculated. The last two formulas we need for our calculations are given below.

$$\begin{aligned}
S &= \mathbb{T}e^{-i\int d^4x \mathcal{H}_I(x)} \\
\mathcal{H}_I(x) &= -\mathcal{L}_I(x)
\end{aligned} \tag{8}$$

In the first equation, $\mathbb{T}$ is our time-ordering product (different from the T-matrix mentioned above), which is not necessary to elaborate on, and $\mathcal{H}_I(x)$ is the interaction Hamiltonian density. The second equation shows how $\mathcal{H}_I(x)$ is related to $\mathcal{L}_I(x)$, the interaction Lagrangian density. This tells us that once we have $\mathcal{L}_I(x)$, we can expand S and calculate each term perturbatively. The exact order of our calculations is as follows.

- We start from a Lagrangian, $\mathcal{L}$, which serves as our model.
- We calculate $\mathcal{M}$ up to a certain order, usually 2^{nd} or 3^{nd} order in the coupling constant.
- We expand the last line of Eq. (6) to perform our quantum calculation of the impulse.
- Finally, we take the classical limit, $\xi, \hbar \to 0$.

The models we have used so far include:

Model 1

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{2}(\partial_\mu A)^2 - \frac{1}{2}gA\phi^2 - \frac{1}{2}gA_{cl}\phi^2 \tag{9}$$

Model 2

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi + ieA^\mu(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*) + e^2 A^\mu A_\mu \phi^* \phi - V(\phi) \tag{10}$$

Model 3

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{2}(\partial_\mu A)^2 - \frac{1}{2}gA\phi^2 - V\phi^2 \tag{11}$$

For Model 1, we started with a a scalar field electron (ϕ) and a scalar field photon (A), which is implied in the first three terms of the Lagrangian. The last two terms characterize the interaction between the fields, where $\frac{1}{2}gA\phi^2$ is the interaction between the ϕ field and the A field and $\frac{1}{2}gA_{cl}\phi^2$ is the interaction between the ϕ field and a classical field, A_{cl}, that we can control. Here, g is our coupling constant, which allows us to solve $\mathcal{M}$ in orders of g. Note, A_{cl} is directly related to our external force; thus, we want to match it to the external force in our quantum calculation.

Model 2 was motivated by the Lagrangian for scalar QED. The main idea is that A^μ is now the classical external field but is also a four vector field. Additionally, e is our coupling constant. The motivation for why we considered Model 2 will become clearer when I discuss my calculations.

Finally, Model 3 is similar to Model 1. We simply replaced $\frac{1}{2}gA_{cl}$ with V to separately characterize the perturbation of our classical field from that of our A field. Hence, V acts as both the coupling constant and the classical field.

3 Calculations

To first motivate why we need to calculate the scattering amplitude $\mathcal{M}$, we will show the full form of $\langle \Delta p^\mu \rangle$. It can be shown that from Eq. (6) and using $S = 1 + iT$, one can derive two possible equations for $\langle \Delta p^\mu \rangle$.

$$\langle \Delta p^\mu \rangle = i \langle \psi | [\mathbb{P}, T] | \psi \rangle + \langle \psi | T^\dagger [\mathbb{P}, T] | \psi \rangle \tag{12}$$

$$\langle \Delta p^\mu \rangle = i \langle \psi | (\mathbb{P}T - T^\dagger \mathbb{P}) | \psi \rangle + \langle \psi | T^\dagger \mathbb{P} T | \psi \rangle \tag{13}$$

Let's call the first term in Eq. (12) $\langle \Delta p^\mu \rangle_{(1)}$ and the first term in Eq. (13) $\langle \Delta p^\mu \rangle_{(3)}$. The second term in both equations is a higher order term that takes into account radiation emitted in the scattering, but we mainly looked at the first term in both equations. After substituting $|\psi\rangle$ from Eq. (5), one can obtain

$$\langle \Delta p^\mu \rangle_{(1)} = i \int (dp)(dp')\psi(p)\psi^*(p')(p' - p)^\mu \mathcal{M}(p \to p') \tag{14}$$

$$\langle \Delta p^\mu \rangle_{(3)} = i \int (dp)(dp')p'^\mu [\psi(p)\psi^*(p')\mathcal{M}(p \to p') - \psi^*(p)\psi(p')\mathcal{M}^*(p \to p')] \tag{15}$$

Our p' comes from $\langle \psi |$, which follows from the formalism in KMOC [2]. In this form, we can see that we need to calculate scattering amplitude first before calculating the expected impulse.

3.1 Feynman Diagrams

Scattering amplitudes are calculated by expanding in orders of our coupling constant and calculating the contribution to each order separately. These contributions can be summarized in terms of Feynman Diagrams (FDs), which also aid in giving us physical intuition for our scattering process. FDs can be different for different scattering processes as well. For example, we looked at the scattering of $\phi(p) \to \phi(p')$ (corresponding to $\mathcal{M}(p \to p')$) and $\phi(p) \to \phi(p') + A(q)$ (corresponding to $\mathcal{M}(p \to p' + q)$). Although the terms in Eqs. (14) and (15) do not contain $\mathcal{M}(p \to p' + q)$, we still computed these FDs for future calculations. These are shown below.

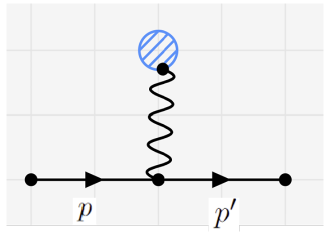

Fig 1: $\mathcal{O}(g^1)$ for Model 1 - $\mathcal{M}(p \to p') = -ig\tilde{A}_{cl}(p'-p)$

$\mathcal{O}(e^1)$ for Model 2 - $\mathcal{M}(p \to p') = -ie\tilde{A}^{\mu}_{cl}(p'-p)(p'+p)_{\mu}$

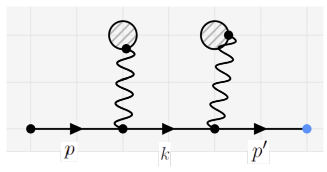

Fig 2: $\mathcal{O}(g^2)$ for Model 1 - $\mathcal{M}(p \to p') = (-ig)^2 \int \frac{d^4k}{(2\pi)^4} \frac{i\tilde{A}_{cl}(p'-k)\tilde{A}_{cl}(k-p)}{k^2-m^2+i\varepsilon}$

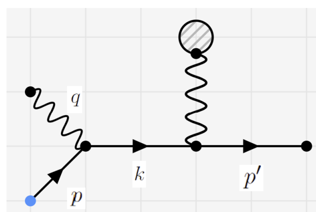

Fig 3: $\mathcal{O}(g^2)$ for Model 1 - $\mathcal{M}(p \to p'+q) = \frac{(-ig)^2}{2} \frac{i\tilde{A}_{cl}(p'+q-p)}{(q-p)^2-m^2+i\varepsilon}$

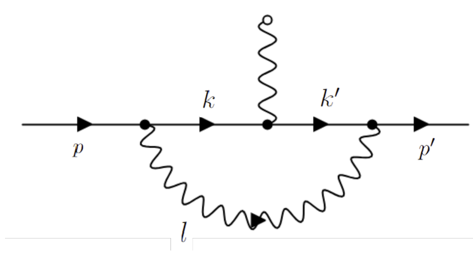

Fig 4

The expression for $\mathcal{O}(g^2V)$ from Model 3, displayed in Fig. (4), is lengthy, so it is expressed separately below. Note, that this is still a 3rd order calculation in $\mathcal{M}(p \to p')$, where V acts as our 3rd coupling constant.

$$\mathcal{M}(p \to p') = (-i)^3 g^2 \tilde{V}(p'-p) \frac{1}{(4\pi)^2} \int_0^1 du_1 \int_0^1 du_2 \frac{-1}{(u_2 p + (1-u_1-u_2)p')^2}$$

The following are a couple things to note in our diagrams.

- The shaded bubbles in the FDs represent our classical field A_{cl} (or A^{μ}_{cl} for Model 2).
- The smaller unshaded bubble represents V for Model 3 in the last diagram.
- The diagrams are intuitive; for example, the FD at $\mathcal{O}(g^1)$ shows how the particle enters with momentum p, interacts with the classical field, and leaves with momentum p'; note that the 'squiggly' lines without the bubbles represents a scalar photon.
- Finally, $\tilde{F}(k)$ represents the Fourier transform of some function $F(x)$.

3.2 Computing $\langle \Delta p^{\mu} \rangle$

Now, we can begin matching the classical impulse with the impulse from the quantum calculation. Here, we will demonstrate the calculations we have done because, as mentioned before, we have not yet been able to match the external field to the corresponding external force in Eq. (3). The order of these quantum calculations are as follows.

- We first start with one of our models, the Lagrangians.

- Next, we choose a classical field and take the Fourier transform of the field.
- Then, we plug in our classical field into the expressions we obtained from our FDs.
- We finally perform the quantum calculations up to the order we are interested in and take the classical limit ($\xi, \hbar \to 0$).

We first started with Model 1, since it is a simple Lagrangian containing scalar fields. We constructed a classical field only dependent on time, x^0, given by

$$A_{cl}(x) = A_{cl}(x^0) = C[\Theta(x^0 + t) - \Theta(x^0 - t)] \ . \tag{16}$$

Here, C is just some constant. We defined our Fourier transforms to be

$$\tilde{f}(q) = \int d^4x f(x) e^{iq\cdot x} \tag{17}$$

$$f(x) = \int \frac{d^4q}{(2\pi)^4} \tilde{f}(q) e^{-iq\cdot x} \ . \tag{18}$$

With these definitions, we obtained

$$\tilde{A}_{cl}(q) = 2C(2\pi)^3 \delta^3(\vec{q}) \frac{sin(q^0 t)}{q^0} \ . \tag{19}$$

Plugging in this expression for $\mathcal{O}(g^1)$ and substituting the resulting expression for $\mathcal{M}(p \to p')$ into Eq. (14) yielded

$$\langle \Delta p^\mu \rangle_{(1)} = i \int (dp)(dp')\psi(p)\psi^*(p')(p'-p)^\mu(-ig)2C(2\pi)^3\delta^3(\vec{p}' - \vec{p})\frac{sin((p'^0 - p^0)t)}{p'^0 - p^0} \ . \tag{20}$$

Recall that

$$(dp) = \frac{d^4p}{(2\pi)^4}(2\pi)\delta(p^2 - m^2)\Theta(p^0) \ .$$

As a result of doing the calculation over the two delta functions from (dp) and (dp') along with the three delta functions from $\tilde{A}_{cl}(p'-p)$, we obtained $\langle \Delta p^\mu \rangle_{(1)} = 0$. This was quite puzzling, as one would expect that if we turn on a constant-nonzero classical field at some time $-t$ and take it off at some time t, we should see some expected change to the momentum of our particle. We also performed the same calculation for the $\mathcal{O}(g^2)$ from Fig. 2 and got 0 again.

At this point, we decided to look at the classical impulse to see if we do get a nonzero answer. We looked at a Lagrangian without our scalar photon, since we were only interested in the scattering of the particle with the classical field,

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}gA\phi^2 \ . \tag{21}$$

Here, $A_{cl} = A$. From Eq. (21), Dr. Luty performed the calculations to go from the Lagrangian to a Schrodinger equation and then to our classical equations of motion. I will not show those calculations here, as they are not necessary to the main idea of this paper. Below are the Schrodinger equation and the corresponding classical equation of motion for our field theory.

$$i\partial_t \Psi(\vec{x}, t) = \left[-\frac{\vec{\nabla}^2}{2m} + \frac{g}{2m}A(\vec{x}, t) \right] \Psi(\vec{x}, t) \tag{22}$$

$$V(\vec{x}, t) = \frac{g}{2m}A(\vec{x}, t) \tag{23}$$

$$\Rightarrow \frac{d^2}{dt^2}\vec{x} = -\vec{\nabla}V = -\frac{g}{2m}\vec{\nabla}A(\vec{x}, t) \tag{24}$$

In Eq. (23), we identify the potential energy from Eq. (22), which results in Eq. (24) since we know that the force is the negative gradient of the potential. From Eq. (24), we can see that our classical impulse will also be 0 since we chose a potential that only depends on time.

This motivated us to look at a model similar to Scalar QED, which resulted in Model 2. We want to keep our force only dependent on time, so that matching the two impulses is simple, at least for our initial calculations. Note that A^μ_{cl} is a vector field for Model 2. We chose a simpler external field this time, given by

$$A^\mu_{cl}(x) = A_0 \Theta(x^0)\hat{z} \ . \tag{25}$$

Here, A^μ_{cl} points in the z direction and is turned on at time $x^0 = 0$. The Fourier transform is given by

$$\tilde{A}^\mu_{cl}(q) = A_0(2\pi)^3\delta^3(\vec{q})\frac{-i\hat{z}}{q^0 + i\varepsilon} \ . \tag{26}$$

We checked the classical equations of motion first, which relies on the normal definitions of the electric and magnetic fields.

$$\vec{E} = -\vec{\nabla}V - \partial_t\vec{A} \ \Rightarrow \ \vec{E}(x^0) = A_0\delta(x^0)\hat{z} \tag{27}$$

$$m\ddot{\vec{x}} = eA_0\delta(t)\hat{z} \ \Rightarrow \ \Delta\vec{p} = eA_0\hat{z} \tag{28}$$

As shown in Eq. (28), our classical impulse is nonzero, so we expect the impulse from our quantum calculation to also be finite and nonzero. For this calculation, we used $\langle\Delta p^\mu\rangle_{(3)}$, which is a similar term to $\langle\Delta p^\mu\rangle_{(1)}$ but easier to use since it involves less momenta. For example, in $\langle\Delta p^\mu\rangle_{(1)}$, we have the term $(p' - p)^\mu$, which could involve two separate calculations. But for $\langle\Delta p^\mu\rangle_{(3)}$, we only need to calculate the first term since the second term is just the conjugate of the first (i.e. real parts cancel and imaginary parts add). In addition, we only looked at the 1st order case $\mathcal{O}(e^1)$ from Fig. 1. The following below are the main points we want to highlight.

$$\langle\Delta p^\mu\rangle_{(3)} = i\int(dp)(dp')p'[\psi(p)\psi^*(p')\mathcal{M}(p\to p') - \psi^*(p)\psi(p')\mathcal{M}^*(p\to p')] \tag{29}$$

$$\begin{aligned}\langle\Delta p^\mu\rangle_{(3),1st} = i\int(dp)&\frac{d^4p'}{(2\pi)^4}(2\pi)\delta(p'^2 - m^2)\Theta(p'^0)p'^\mu\psi(p)\psi(p')\\ &\times(-ie)A_0(2\pi)^3\delta^3(\vec{p}' - \vec{p})\frac{-i}{p'^0 - p^0 + i\varepsilon}(p'_3 + p_3)\end{aligned} \tag{30}$$

$$\langle\Delta p^\mu\rangle_{(3),1st} = \frac{1}{2}eA_0\frac{m\xi}{\varepsilon}\hat{z} \tag{31}$$

We first wrote down Eq. (15) again for reference, and $\langle\Delta p^\mu\rangle_{(3),1st}$ corresponds to the 1st term in $\langle\Delta p^\mu\rangle_{(3)}$. We substituted in the expression for the scattering amplitude at $\mathcal{O}(e^1)$ and wrote out the expression for (dp') in Eq. (30) to show that the delta function from (dp') in conjunction with the delta functions from A^μ_{cl} forces $p'^\mu = p^\mu$. The resulting expression is in Eq. (31).

Although this expression looks similar to the classical impulse in Eq. (28), note that $1/\varepsilon$ shows up in our calculations. Previously in our Fourier transform, Eq. (26), a factor of $+i\varepsilon$ is added to the denominator to make our transform well-defined. But this ε is an infinitesimal, which means that, we must take it to 0 at the end of our calculation. One may also think that since we are taking $\xi \to 0$, maybe we just need to simplify the expression more. However, ξ is not an infinitesimal like ε. When we take the limit as $\xi \to 0$, we are looking for an expression that can be represented in powers of ξ (similar to taking the first few terms of a Taylor series for a function of a small argument). Therefore, ξ is not necessarily 0 but ε is. Consequently, $\langle\Delta p^\mu\rangle_{(3),1st} \to \infty$.

4 Discussion

We have done other calculations, using different classical fields, but we have yet to obtain a finite, non-zero term. Most of our calculations result in 0 with the rest being infinities. Having discussed these problems with Dr. Luty, we came up with a couple ideas to figure out the root of this problem.

In general, one simple idea is that we should maybe look to higher orders with a scalar particle scattering into a scalar particle while emitting a scalar photon. These types of interactions are characterized by the FDs Fig. 3 and Fig. 4. Dr. Luty also proposed that we look at the classical equations of motion once more to come up with a suitable classical field that we can match on to our quantum calculation; however, we are still working out the details of this. I am very interested in figuring out what the KMOC formalism is doing to my classical field, so I have been looking into calculating the $\langle\Delta p^\mu\rangle$ for (1) and (3) without choosing a particular field. The calculation is quite extensive, but I am currently at the point where I need to make the right approximations, which seems promising at first glance. It might take some time before we see any plausible results because of my other commitments at Florida State University, but I expressed my interest to continue this project with Dr. Luty, with which he was pleased to do.

5 Conclusion

To summarize, our project is focused on calculating higher order terms to the AL force, such that it matches on to the terms obtained in Eq. (3). Currently, we are still trying to match our calculations to the first term.

Some of the overall benefits of this project include

- clearing up any confusion about radiation reaction
  - Radiation reaction is a commonly confusing topic in advanced electrodynamics.
  - This is because the AL force term has to be there due to conservation of energy and momentum, but it gives you nonphysical solutions to the motion of point particles.
  - Our model, which we want to derive from QFT, should tell us that the AL force will always have a small effect on our particle; hence, the runaway solutions will be absent.
- and allowing precise calculations of accelerating charged particles.
  - For experimentalists working with accelerating charged particles, our model should allow them to calculate the motion of their particles to as much precision as needed.
  - For example, this can be useful in collision experiments.

6 Acknowledgements

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References

[1] Mario D'Andrea, Markus A. Luty, and Christopher B. Verhaaren. Radiation reaction, over-reaction, and under-reaction. *arXiv (Cornell University)*, January 2019.

[2] David A. Kosower, Ben Maybee, and Donal O'Connell. Amplitudes, observables, and classical scattering. *Journal of High Energy Physics*, 2019(2), February 2019.