# An estimate of uncorrelated production of double quarkonium in PbPb collisions at $\sqrt{s_{NN}} = 5.02 \text{ TeV}$

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We estimate the uncorrelated production of double quarkonium ( $\Upsilon$  and  $J/\psi$  mesons) in PbPb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV. We use a standard Monte Carlo Glauber model to simulate independent nucleon-nucleon collisions. We use PYTHIA 8.306, a Monte Carlo particle production simulation, to obtain quarkonium kinematics including decay into dimuons. Finally, a multinomial model, using probabilities obtained from relevant literature as well as from PYTHIA, was used to obtain estimates of double quarkonium yields corresponding to an integrated luminosity of  $\mathcal{L}_{int} = 1.7$  nb<sup>-1</sup> collected by CMS in the 2018 PbPb run. Distributions from PYTHIA for the  $\Upsilon$  and  $J/\psi$  mesons and their daughter muons were used to calculate acceptance, and muon efficiencies were obtained from studies done in CMS. Estimates for the probability of production for the  $\Upsilon$  and  $J/\psi$  mesons were made from these values. The estimates of double quarkonium production can be used for comparison to measurements currently being carried out by the CMS collaboration.

Keywords: Heavy ion collisions, quarkonium, double quarkonium production, CMS, LHC

# I. INTRODUCTION

The study of quarkonium production has long been considered an excellent method in exploring rare QCD phenomena [1]. In particular, quarkonium production, i.e. heavy meson  $(\mathcal{Q}\overline{\mathcal{Q}})$  production, in heavy ion collisions has historically been used as a signature for the quarkgluon plasma (QGP), a state of matter where, at high enough energies and/or densities, quarks become deconfined [2–4]. The QGP is similar to the electromagnetic plasma present in the sun; however, this is a plasma involving the strong force, i.e. QCD [3, 5]. We use guarkonium as a signature of the QGP for several reasons. First, quarkonium is produced in both proton-proton (pp) collisions and heavy ion (HI) collisions. This means that as a baseline we have production of quarkonium in pp collisions. Furthermore, a QGP is not created in pp collisions whereas there is a large body of research indicating that the QGP is created in PbPb collisions [2]. Finally, as quarkonium is produced in the early stages of a heavy ion collision, suppression due to the formation of a quark gluon plasma will be easily seen [6-10].

Important to our research is the concept of *quarkonium* suppression in the presence of a QGP. This is due to Debye screening, or more specifically, color Debye screening. Screening behaves on the same principles as in electromagnetism, except with color charge. This means, if a quarkonium is produced and then placed in a QGP, its component quarks can become dissociated. If this is the case, then during hadronization—formation of hadrons from a QGP—the component quarks of a heavy quarkonium state within the QGP can make a bound state with another, lighter quark in the plasma thus suppressing the ultimate production of the original quarkonium. As a QGP is formed in heavy-ion collisions, we should see the effects of suppression in quarkonium production.

Single production of quarkonium is considered a rare process in collisions of heavy ions and has been rigorously studied at the Large Hadron Collider (LHC) at CERN and the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Lab. Most particles produced both in HI and pp collisions are  $\pi^+, \pi^-$  and other lighter particles, while heavy mesons are not as readily produced even when suppression is not considered. Subsequently, to study double production makes a rare process even rarer. The double production mechanism is, however, not rigorously modeled in heavy-ion collisions. In pp collisions, double productions can arise as a result of double parton scattering (DPS) [1, 11, 12]. This is indeed very rare in pp collisions; however, is theoretically much less rare in HI collisions as there are far more partons involved in each event. As a consequence, we should see an enhancement of double productions in HI collisions as apposed to pp collisions. Conversely, as discussed above, suppression plays a large role in HI collisions so we should actually see fewer productions in HI as apposed to in pp. This tension between enhancement and suppression has not been extensively studied and as such we do not know what affect the QGP will have in HI collisions. In this paper we will discuss a simple model for uncorrelated production of bottomonium (bb mesons), specifically the  $\Upsilon(1S)$ , and charmonium ( $c\bar{c}$  mesons), specifically  $J/\psi(1S)$ , pairs in order to gain a better understand of the double production mechanism. Our model will not include suppression due to the QGP and, as such, will serve as a baseline for analysis of the QGP.

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This research is being done to provide a model for double production for comparison to experimental data from the Compact Muon Solenoid (CMS) at the LHC, from the 2018 PbPb run at  $\sqrt{s_{NN}} = 5.02$  TeV. The comparison currently being used for data analysis is from d'Enterria and Snigirev, 2014 [11]. The results obtained in the 2014 paper do not currently match preliminary experimental data. Thus, our analysis will provide yet another benchmark for data analysis currently being done at the University of California, Davis as part of the CMS collaboration.

## II. METHODS AND MODELING

In order to be able to experimentally validate our model, we must consider how our detector operates. We first need some mechanism to reconstruct each particle, in other words, we need some way to see each  $\Upsilon$  and  $J/\psi$  meson. We use the decay channel  $\Upsilon \rightarrow \mu^+ + \mu^$ and  $J/\psi \rightarrow \mu^+ + \mu^-$  which have branching ratios of 2.5% and 6% respectively. While this is, of course, not the only way in which our mesons can decay, it is the easiest decay channel to see with our detector and each have sufficiently high branching ratios in order to observe significant meson production. Once we have a production mechanism, we then need a model to find the relative production of each pair.

TABLE I. Glauber parameters used in Glauber model. Values correspond to energy of  $\sqrt{s_{NN}} = 5.02$  TeV.

$\sigma^{pp}_{\rm inel}$	$(70\pm5)\mathrm{mb}$
a	$0.546\mathrm{fm}$
R	$6.62\mathrm{fm}$

Our model will be based on a Monte Carlo Glauber calculation [13–16]. The Glauber model for modeling heavy ion collisions assumes each nucleus-nucleus collision is a superposition of nucleon-nucleon collisions. This means that we treat each collision as a proton-proton collision, as at high energies protons and neutrons are indistinguishable. Glauber parameters used in our simulation are provided in Table I. By randomly throwing ten million simulated collisions, we can obtain several useful parameters. Using our simulation we can determine both the total number of events at  $\int \mathcal{L} dt = 1.7 \,\mathrm{nb^{-1}}$ , where  $\mathcal{L}$  is the luminosity, and obtain a distribution for the number of collisions,  $N_{coll}$ , from which to compute probabilities. We also need to determine the PbPb cross-section,  $\sigma^{PbPb}$ , which can be calculated directly from impact parameter. Using our impact parameter distribution, seen in Fig. 1, we can make a cut at  $b_{lim}$  somewhere before the distribution drops off. If we consider two hard spheres with combined radius  $b_{lim}$ , then their cross-section would simply be  $\sigma = \pi \cdot b_{lim}^2$ . We can do the same thing with the integral of our impact parameter distribution and then



FIG. 1. Impact Parameter distribution normalized to the PbPb cross-section.

scale by our arbitrary cut, thus finding a value for the total PbPb cross-section,  $\sigma^{PbPb}$ . Importantly, from  $\sigma^{PbPb}$ we can determine the number of events at a certain integrated luminosity by

$$N_{events} = \sigma^{PbPb} \cdot \mathcal{L}.$$
 (1)

For our model of production, we assume that for every collision of nucleons, there is some probability of producing a rare particle. Thus, we need to know how the number of collisions is distributed for our total number of events. This distribution can be seen in Fig. 2. We can then use our assumption of production to make an estimate for the number of quarkonium produced in a single Monte Carlo event. By simulating many such events, we can obtain the total expected quarkonium yield in a given integrated luminosity. This prediction is based on our calculation of the probability of production.

#### A. Probabilities

In order to calculate our probabilities, we use the assumption that the probability of production in a particular nucleon-nucleon collision is given by

$$Pr(\mathcal{Q}) = \frac{\sigma^{\mathcal{Q}}}{\sigma^{pp}} \tag{2}$$

where Q represents our quarkonium and  $\sigma$  is our crosssection. The pp cross-section is given by the Glauber



FIG. 2. Distribution for number of collisions from Glauber model.

parameters we used, which is an appropriate cross section for energies of  $\sqrt{s_{NN}} = 5.02$  TeV. We obtain values for  $J/\psi$  and  $\Upsilon$  cross-sections from relevant literature [11, 12, 17]. We must then make sure our values for cross-section take into account the phase-space covered by the detector. In CMS we can only cover the pseudo-rapidity region  $|\eta| < 2.4$  for each single muon. Additionally, in order to compare to data, we must only include prompt production including contribution from feed-down and we must consider the branching ratio for our selected production mechanism. This is summarized in the below equation

$$Pr = BR \times \left. \frac{d\sigma^{\mathcal{Q}}}{d_y} \right|_{y < 2.4} \cdot \frac{1}{F^{\text{direct}} \cdot \sigma^{pp}} \tag{3}$$

where Pr is the probability of production, BR is the branching ratio, and  $F^{direct}$  is the scaling parameter to go from direct to prompt production. Scaling values of cross-section appropriately we find values for probability of production given in Table II.

TABLE II.  $\Upsilon$  and  $J/\psi$  cross-sections at  $\sqrt{s_{NN}} = 5.02$  TeVand their corresponding probabilities.

	Υ	$J/\psi$
$BR \times \text{Prompt } d\sigma^{\mathcal{Q}}/dy _{y < 2.4}$	$9.40\mathrm{nb}$	$2.65\mu{ m b}$
Probability	$1.34 \times 10^{-7}$	$3.78 \times 10^{-5}$

where "Probability" is the probability of producing a quarkonium in a single nucleon-nucleon collision at  $\sqrt{s_{NN}} = 5.02$  TeV and  $\mathcal{L}_{int} = 1.7$  nb<sup>-1</sup>.

## B. Probability Model

To model production we will use a multinomial distribution. In order to verify our model matches data, we perform first many Monte Carlo pseudo-experiments on single quarkonium production and compare this to a binomial distribution. In our model we define "success" as the production of a quarkonium and "failure" as production of no quarkonium. Note that a pseudo-experiment is one set of "throws" of random numbers to determine if a certain event passes a defined threshold or not. In order to determine if this method works at various probabilities, we use successively lower probabilities of production and compare our binomial with a Monte Carlo as in Fig. 3. We find that the two methods agree and thus expand our binomial to a multinomial distribution. We note that the binomial distribution is a special case of the multinomial and define "success" and "failure" as above. The functional form of the multinomial distribution for no rare productions is

$$Pr(X = x) = N_{coll} \cdot p_1^{N_{coll}} \cdot p_2^0 \cdot p_3^0,$$
(4)

for single production is

$$Pr(X = x) = N_{coll} \cdot (N_{coll} - 1) \cdot p_1^{N_{coll} - 1} \cdot p_2^1 \cdot p_3^0, \quad (5)$$

for double production of same quarkoniun is

$$Pr(X = x) = \frac{N_{coll}}{2} \cdot (N_{coll} - 1) \cdot p_1^{N_{coll} - 2} \cdot p_2^2 \cdot p_3^0, \quad (6)$$

and for double production of differing quarkonium is

$$Pr(X = x) = N_{coll} \cdot (N_{coll} - 1) \cdot p_1^{N_{coll} - 2} \cdot p_2^1 \cdot p_3^1 \quad (7)$$

where  $N_{coll}$  is the number of trials,  $p_2$  and  $p_3$  are the probabilities of  $\Upsilon$  and  $J/\psi$  respectively, and  $p_1 = 1-p_2 - p_3$ . After completing another set of Monte Carlo pseudoexperiments and comparing these to the multinomial, we can verify our multinomial model. Note that, however, this model only agrees with a Monte Carlo at probabilities of production below  $10^{-3}$  as above this threshold triple production and above significantly contribute to the overall production. Finally, we apply the probabilties calculated in Table II to obtain realistic values for production.

## C. Acceptance and Efficiency

With the above calculated probabilities we can find the production of our mesons without taking into account considerations of our detector as in panel (a) of Fig. 5. Of course, in order to experimentally verify our





FIG. 3. Comparison of Monte Carlo and binomial distribution at successively lower probabilities.

results we need to scale these results to include the acceptance  $(\mathcal{A})$  and efficiency  $(\mathcal{E})$  of our detector. Acceptance is defined as the number of muon pairs (i.e. decays from an  $\Upsilon$  or  $J/\psi$ ) that land in the kinematics range of the relevant detectors—such as the tracker and muon chambers of CMS—divided by the total number of  $\Upsilon$  or  $J/\psi$ produced. In order for a muon to be detected, it needs enough energy to reach the detector and it needs to land within the detector range. In particular, the muon chambers need high transverse momentum because they are positioned at large radius and outside of the solenoidal magnet. For CMS, each muon needs a certain amount of energy and needs to land in  $|\eta| < 2.4$  to be accepted [18]. For the  $\Upsilon$  we use  $p_T > 3.5 \,\text{GeV}$  and for the  $J/\psi$  we use published values of acceptance from CMS [19]. After being accepted by the detector, there is some probability of the meson itself being reconstructed. This variable is called *efficiency* and is defined by the number of reconstructed  $\Upsilon$  or  $J/\psi$  mesons divided by the total number of  $\Upsilon$  or  $J/\psi$  mesons produced. Unfortunately, we cannot experimentally find the total number of mesons for the denominator of our acceptance or efficiency. Thus, we need a simulation to produce our rare particles and allow only dimuonic decay.

We use the Monte-Carlo particle production simulation, PYTHIA 8.306 [20, 21], to simulate production of  $\Upsilon$  and  $J/\psi$  mesons. This simulation allows us to require decay into dimuons. Simulating one million events for each of our mesons from all 1S production mechanisms we can find a distribution for transverse momentum,  $p_T$ , and pseudorapidity,  $\eta$ , over all phase space [22–24]. Making kinematic cuts as defined above we find values for acceptance. In order to calculate efficiency we need to

TABLE III. Values of acceptance and efficiency calculated from PYTHIA simulation.

	Υ	$J/\psi$
Acceptance	0.25	0.014
Efficiency	0.80	0.67
Combined	0.20	0.0094

perform a Monte-Carlo simulation using a probability map of CMS efficiency values [25]. By throwing each muon pair from PYTHIA into  $p_T$ ,  $\eta$  space one thousand times and using CMS data to determine probability of reproduction, we can find average values for efficiency. These values can be found in Table III, and a visualization of this experiment can be found in Fig. 4. Panel (a) shows the pseudo-experiment distribution for 1000 pseudo-experiments using the one million generated  $J/\psi$ while (b) shows the distribution for the  $\Upsilon$ . Each is fit with a gaussian to verify that a simple average will suffice for determining a single value of  $\mathcal{A} \times \mathcal{E}$ . In both cases our low  $\chi^2$  values verify the gaussian fit, and the resulting distributions demonstrate the successful completion of both the PYTHIA simulation as well as the followup Monte Carlo pseudo-experiments in ultimately determining acceptance and efficiency and hence real-world production.

We finally note that we have not included nuclear modification factors due to suppression such as  $R_{AA}$ . This means our model will not account for suppression due to the QGP but will rather serve as a benchmark in double production.

#### **III. RESULTS AND DISCUSSION**

Combining all of our above results we can finally obtain values for production of our quarkonia. These results are shown in Fig. 5. In panel (a), uncorrelated production is shown without accounting for acceptance and efficiency loss. This is, essentially, the total production of the various single and double quarkonium events given the CMS running conditions in the 2018 PbPb run at  $\sqrt{s_{NN}} = 5.02 \text{ TeV}, \ \mathcal{L}_{int} = 1.7 \text{ nb}^{-1}.$  In panel (b) we see the total uncorrelated production accounting for acceptance and efficiency losses. This represents what our detector actually sees. Values for production are summarized in Table IV. Immediately apparent is the significant reduction in production from the case with to the case without acceptance and efficiency. In fact, without acceptance and efficiency considered, we note that double  $J/\psi$  production is actually greater than single  $\Upsilon$  production. This drastically changes when our acceptance and efficiency are included. When considering double production we note that we should most prominently see  $J/\psi + J/\psi$  production, although  $\Upsilon + J/\psi$  production also lies within the uncertainty of the  $J/\psi$  double pro-



FIG. 4. Acceptance and efficiency calculations for both the  $\Upsilon$  and  $J/\psi$  using a Monte Carlo approach for finding efficiency. Distribution of pseudo-experiments is fit with a gaussian.

duction. Thus, when looking for double production, we recommend looking for this production channel.

In our analysis we did come across some discrepancy in estimates of cross-section and hence drastically differing values of probability. In order to obtain a verification of the probability values quoted above, we compare our single production values to data. We first include suppression by multiplying by  $R_{AA}$  values quoted from relevant literature. For rough estimates of production with suppression we use  $R_{AA} \approx 0.38$  for  $\Upsilon$ , and  $R_{AA} \approx 0.35$  for  $J/\psi$  [6, 7, 26]. Incorporating these values in our model we compare what our model indicated single production should be to single production calculated from data by the CMS group at UC Davis. For the  $\Upsilon$  data, we use uncorrelated yields of  $\Upsilon$  from the same  $\sqrt{s_{NN}} = 5.02$  TeV data set as above. For  $J/\psi$  we use uncorrelated single yields from pPb data at  $\sqrt{s_{NN}} = 8.16$  TeV and compare this to  $\sqrt{s_{NN}} = 5.02$  TeV. For both single  $\Upsilon$  and single  $J/\psi$  production, our model is consistent with data.

TABLE IV. Results of quarkonium yields with and without accounting for  $\mathcal{A}\times\mathcal{E}$  losses

	Without $\mathcal{A} \times \mathcal{E}$	With $\mathcal{A} \times \mathcal{E}$
None	$1.25 \times 10^{10} \pm 4.6 \times 10^{6}$	$1.27 \times 10^{10} \pm 4.7 \times 10^{6}$
$J/\psi$	$1.85 \times 10^8 \pm 5.6 \times 10^5$	$1.76\times10^6\pm5.5\times10^4$
Υ	${6.58\times10^{5}\pm3.36\times10^{4}}$	$1.32\times10^5\pm1.5\times10^4$
$J/\psi + J/\psi$	${3.82\times10^{6}\pm8.09\times10^{4}}$	$341\pm770$
$J/\psi+\Upsilon$	$2.72\times10^4\pm6.82\times10^3$	$51\pm297$
$\Upsilon+\Upsilon$	$48\pm287$	$2\pm57$

## IV. CONCLUSIONS

We model the uncorrelated production of single and double quarkonia ( $\Upsilon$  and  $J/\psi$  mesons) in PbPb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV. Our model includes a Monte Carlo Glauber model to obtain the number of nucleonnucleon collisions; various cross-sections for probability of quarkonium production; a multinomial model to account for these probabilities and estimate production; and a PYTHIA simulation to determine kinematic cuts to produce yield results comparable to data. The purpose of this research is to obtain theoretical values for double production. These values will be compared to experimental data from the CMS 2018 PbPb run at the same energy as above currently being analyzed by the nuclear group at the University of California, Davis as part of the CMS experiment. Preliminary results indicate that, for instance, double production of  $\Upsilon + J/\psi$  lie around five total productions for this run. Our analysis will provide a crucial benchmark in determining the effect the QGP plays in quarkonium production. Of course, as our model does not include suppression, experimental data should find production values much lower than we find.

Future work on this model could go into extending the particles considered for double production. For instance, we could include Z and W boson double production in conjunction with  $\Upsilon$  and  $J/\psi$  production. Additionally, we could add a mechanism for correlated production. While correlated production has been observed to contribute to production by nearly a factor of 200 less than uncorre-

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FIG. 5. Visualization of final results. Panel (a) shows values without acceptance and efficiency whereas (b) shows values with acceptance and efficiency.

lated production and would thus be nearly undetectable in our model, adding this mechanism would nonetheless provide meaningful insight to the double quarkonium production mechanism.

Finally, the direct application of the results presented above will be as a comparison to experimental data. This data is currently being analyzed and will be available in the next year. From the experimental results we will be able to infer to some extent what effect the QGP has in the double production of quarkonium.

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