Calculating an Observable for Light Dark Matter

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Abstract

We used scattering theory to calculate a new observable for a light dark matter particle candidate. This observable is the force the dark matter would exert on ordinary matter. We then looked at several force experiments and found that at least one could be sensitive to a range of masses and interaction strengths of the particle that has not yet been excluded.

1 Introduction

Dark matter makes up about 85% of matter in the universe, but very little is known about it [1]. It is possible that dark matter could be explained by revising gravitational theory, but most physicists agree that it is more likely to be explained by some sort of additional matter that does not interact with observable electromagnetic radiation. Furthermore, it is probably not even baryonic.

Assuming that dark matter is composed of non-baryonic particles, the most common model for its distribution is a spherical halo surrounding each galaxy, with the galaxy rotating with respect to the halo [2]. Using this model, we find that the density of dark matter is $\rho = 0.6$ GeV/cm^3, and the velocity of the dark matter in the Milky Way with respect to earth is around $v = 10^{-3}$ c. However, the mass of the dark matter particle is still unknown, as is its interaction with the Standard Model.

One possible solution is light dark matter, which is composed of particles with mass less than 1 eV. Since these particles are so light, there must be greater than 10^9 particles per cm^3 to maintain the known dark matter density ρ. If these particles are spin-0 bosons, then the Pauli exclusion principle does not apply and there can be infinite degeneracy. The particle density combined with infinite degeneracy means that we can use a classical field approximation to describe the particles, with a change to the Standard Model Lagrangian given by

$$\Delta\mathcal{L} = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2 - \frac{1}{f}\phi^2\bar{\psi}\psi), \tag{1}$$

where ϕ is the dark matter field, m is the mass of the dark matter, f determines the strength of interaction between dark matter and ordinary matter, and ψ is a nucleon. This change to the Lagrangian could be significant if there is a $\phi \to -\phi$ symmetry. The goal of this project was to determine the force such a field exerts on ordinary matter and to find experiments that might be sensitive enough to measure the force.

Solving Eq. 1 using the usual Euler-Lagrange equations, we find the equation of motion

$$(\partial_t^2 - \nabla^2 + m^2 + \tfrac{n(x)}{f})\phi = 0, \tag{2}$$

where $n(x) = \langle\bar{\psi}\psi\rangle$ is the average number density of nucleons. The basic setup we considered was having a spherical scattering object in a moving background dark matter field. Using the solution to Eq. 2, we can then calculate the force the dark matter exerts on the scattering object using the energy-momentum tensor, which is given by

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \eta_{\mu\nu}\mathcal{L}, \tag{3}$$

where

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}m^2\phi^2 \tag{4}$$

(note the extra term in Eq. 1 is not present because it is only non-zero inside the scattering object, while the force is exerted from the outside). We are only interested in the real space components of the tensor, so Eq. 3 becomes

$$T_{ij} = \partial_i \mathrm{Re}(\phi)\partial_j \mathrm{Re}(\phi) + \tfrac{1}{2}\delta_{ij}\big[\big(\partial_t \mathrm{Re}(\phi)\big)^2 - \big(\partial_r \mathrm{Re}(\phi)\big)^2 - \big(\tfrac{1}{r}\partial_\theta \mathrm{Re}(\phi)\big)^2 - m^2 \mathrm{Re}(\phi)\big], \tag{5}$$

where Latin indices indicate values of {1,2,3} rather than the standard {0,1,2,3} and we have used the flat spacetime metric with diagonal $(+,-,-,-)$. Finally, the momentum transferred from the dark matter to the object is given by

$$\frac{dp^i}{dt} = -\int dA^j T_{ij}. \tag{6}$$

This transferred momentum is equal to the force of the dark matter on the object. Due to the symmetry of the system, this force is entirely in one direction, which we choose to be the z-direction. Thus, we only have to calculate the z-component of the force. Writing this in polar coordinates, we have

$$\begin{aligned}\frac{dp_z}{dt} &= -2\pi r^2 \int_{-1}^{1} du \hat{r}_i T_{iz} \\ &= -2\pi r^2 \int_{-1}^{1} du u\Big[\big(1-u^2\big)\frac{1}{r}\partial_u \mathrm{Re}(\phi)\partial_r \mathrm{Re}(\phi) \\ &\qquad + \frac{1}{2}\Big(\big(\partial_t \mathrm{Re}(\phi)\big)^2 - m^2 \mathrm{Re}(\phi)^2 + \big(\partial_r \mathrm{Re}(\phi)\big)^2 - \frac{1-u^2}{r^2}\big(\partial_u \mathrm{Re}(\phi)\big)^2\Big)\Big],\end{aligned} \tag{7}$$

where $u = \cos\theta$ and $\hat{r}_i$ is the cartesian unit vector.

2 Calculating Force

The form of the solution to Eq. 2 is standard from scattering theory [3]. The wavefunction outside the scattering object can be broken up into two parts: an incident plane wave that simply describes the dark matter field and a scattered wave that accounts for the residual effects of the interaction with ordinary matter, i.e.

$$\phi_{r>R}(r) = \phi_{inc}(r) + \phi_{scatt}(r). \tag{8}$$

The incident wave is given by

$$\phi_{inc}(r) = \frac{2\rho v}{k} e^{ikz} e^{-imt} = \sum_\ell \frac{2\rho v}{k}(2\ell+1) i^\ell j_\ell(kr) P_\ell(u) e^{-imt}, \tag{9}$$

the scattered wave is given by

$$\phi_{scatt}(r) = \sum_\ell (A_\ell j_\ell(kr) + B_\ell n_\ell(kr)) P_\ell(u) e^{-imt}, \tag{10}$$

and the wavefunction inside the scattering object is given by

$$\phi_{r<R}(r) = \sum_\ell C_\ell j_\ell(\tilde{k}r) P_\ell(u) e^{-imt}, \tag{11}$$

where r is the distance from the center of the scattering object, $k = mv$, $\tilde{k} = \sqrt{k^2 - \frac{n}{f}}$, j_ℓ and n_ℓ are the standard Bessel and Neumann functions, respectively, and A_ℓ, B_ℓ, and C_ℓ are constants determined by the boundary conditions at infinity, the surface of the scattering object, and the center of the scattering object.

The boundary condition at infinity is that the scattered component of the wave becomes spherically symmetric, i.e.

$$\phi_{scatt}(r) \propto e^{ikr}. \tag{12}$$

Using the large r approximation for the Bessel functions in Eq. 10 and setting the e^{-ikr} component equal to zero produces the constraint

$$B_\ell = iA_\ell, \tag{13}$$

so that Eq. 10 becomes

$$\phi_{scatt}(r) = \sum_\ell A_\ell(j_\ell(kr) + in_\ell(kr))P_\ell(\cos\theta)e^{-imt}. \tag{14}$$

Thus the total wavefunction outside the scattering object is

$$\phi_{r>R}(r) = \sum_\ell \big[\big(\frac{2\rho v}{k}(2\ell+1)i^\ell + A_\ell\big)j_\ell(kr) + iA_\ell n_\ell(kr)\big]P_\ell(\cos\theta)e^{-imt}. \tag{15}$$

The remaining boundary conditions at the surface and center of the scattering object can be used to calculate the values of A_ℓ and C_ℓ. Then we can plug the wavefunction into Eq. 7 to obtain the force of the dark matter on the object.

2.1 Hard Sphere Approximation

To calculate the wavefunction at a finite distance from the scattering object, we began by using the hard sphere approximation, which assumes that the potential is infinite within the scattering object. This means that the wavefunction vanishes at the surface of the object ($r = R$) and is equal to zero within the object. To determine the wavefunction outside the object, we apply this condition to 15, which yields

$$A_\ell = -\frac{2\rho v}{k}(2\ell+1)i^\ell \frac{j_\ell(kR)}{j_\ell(kR) + in_\ell(kR)} \tag{16}$$

so that

$$\phi_{r>R}(r) = \sum_\ell \frac{2\rho v}{k}(2\ell+1)i^\ell \Big[j_\ell(kr) - \frac{j_\ell(kR)}{j_\ell(kR) + in_\ell(kR)}\big(j_\ell(kr) + in_\ell(kr)\big)\Big]P_\ell(\cos\theta)e^{-imt}. \tag{17}$$

Recall that the force of the dark matter on the scattering object is given by Eq. 7. The dark matter exerts a force on the object at the surface of the object, so we only care about the value of the momentum transfer at $r = R$. Since the wavefunction vanishes at $r = R$, only the $(\partial_r\phi)^2$ term will survive in Eq. 7, so we only have to evaluate

$$\frac{dp_z}{dt} = -\pi r^2 \int_{-1}^{1} du u\big(\partial_r \mathrm{Re}(\phi)\big)^2, \tag{18}$$

where ϕ is given by Eq. 17. Evaluating this at $r = R$ and time-averaging yields

$$\langle\frac{dp_z}{dt}\rangle = 4\pi(kR)^2\big(\frac{2\rho v}{k}\big)^2 \sum_\ell (\ell+1)i[N_{+,\ell}(kR)N_{-,\ell+1}(kR) - N_{+,\ell+1}(kR)N_{-,\ell}(kR)], \tag{19}$$

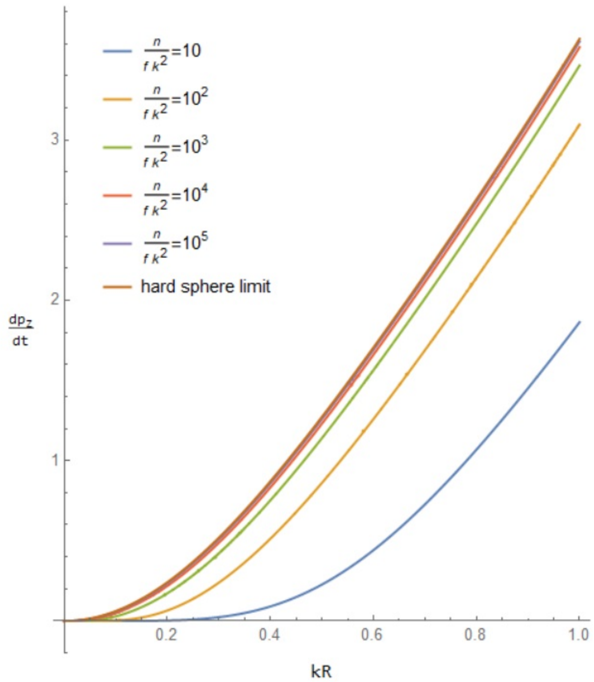
where

$$N_{\pm,\ell}(kR) = j'_\ell(kR) - j_\ell(kR)\frac{j'_\ell(kR) \pm in'_\ell(kR)}{j_\ell(kR) \pm in_\ell(kR)}, \tag{20}$$

as shown in Figs. 1 and 2 for fixed particle mass and object radius, respectively (recall $k = mv$). The momentum transfer increases as the radius of the scattering object increases but decreases as the mass of the dark matter particle increases.

2.2 Finite Potential

To obtain the more general solution, we change the boundary conditions so that the wavefunction and its derivative must be continuous at the surface of the scattering object. Because of this continuity, and because



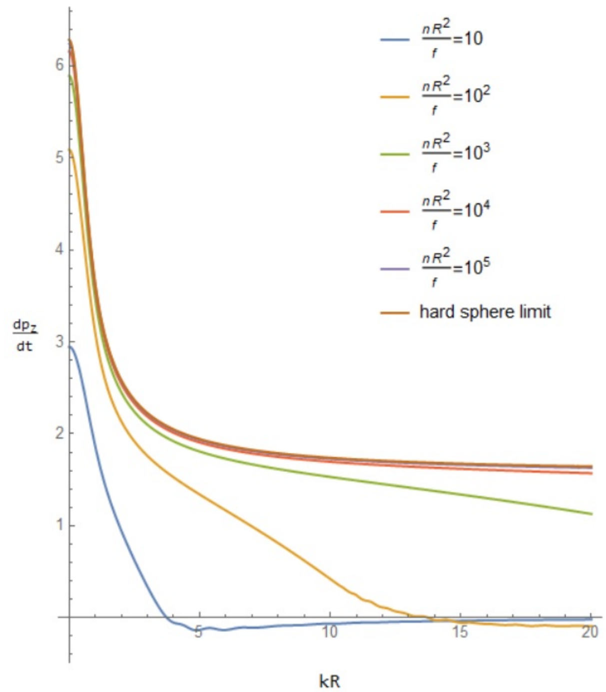


Figure 1: Force versus kR for fixed mass (recall $k = mv$). The force increases as the radius increases, and the force approaches the hard sphere limit as the potential increases.

Figure 2: Force versus kR for fixed radius. The force decreases as the mass increases (recall $k = mv$), and the force approaches the hard sphere limit as the potential increases. This solution is only valid for $k^2 < \frac{n}{f}$.

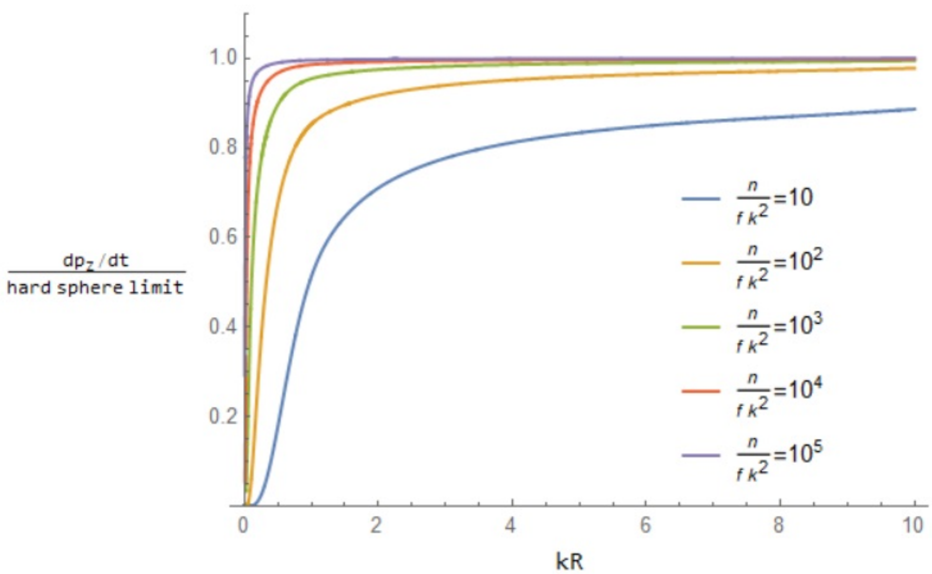


Figure 3: Ratio of force with a finite potential and force in hard sphere approximation. The force approaches the hard sphere limit as the radius of the scattering object increases.

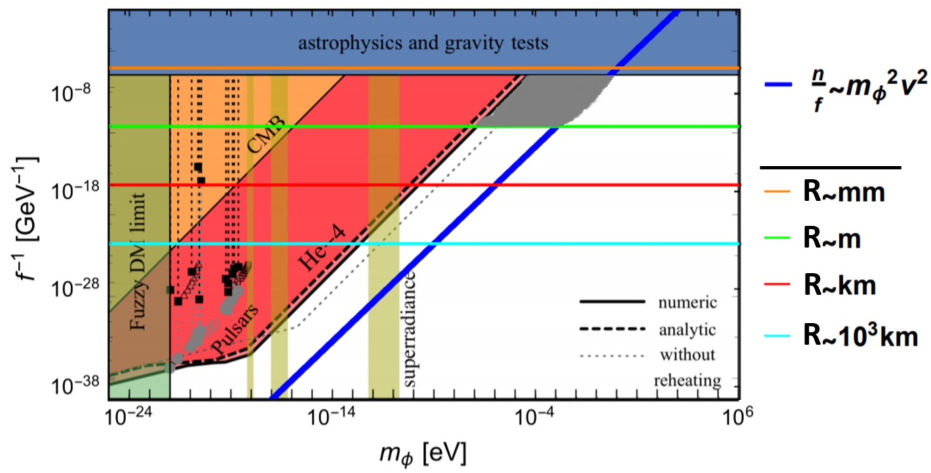


Figure 4: Exclusion plot for light dark matter particle candidate. The region to the left of the diagonal line has a scattering potential. The horizontal lines indicate the radius required to be able to use the hard sphere approximation. The proposed experiment should be sensitive to the region shaded in grey.

we never use more than a single derivative of the wavefunction, we only need to calculate the momentum transfer inside the object before setting $r = R$. Applying the boundary conditions to Eq. 11 and 15 yields

$$C_\ell = \frac{2\rho v(2\ell+1)i^\ell}{k(ia_\ell + b_\ell)}, \tag{21}$$

where

$$a_\ell = k^2 R^2 j_\ell(\tilde{k}R) j_{\ell+1}(kR) - k\tilde{k}R^2 j_{\ell+1}(\tilde{k}R) j_\ell(kR) \tag{22}$$

and

$$b_\ell = k\tilde{k}R^2 j_{\ell+1}(\tilde{k}R) n_\ell(kR) - k^2 R^2 j_\ell(\tilde{k}R) n_{\ell+1}(kR). \tag{23}$$

Then the time-averaged force evaluated at $r = R$ is given by

$$\begin{aligned}\langle\frac{dp_z}{dt}\rangle =& \pi \sum_\ell \frac{\ell+1}{(2\ell+1)(2\ell+3)} \\ & \times [C_\ell C^*_{\ell+1}\big(|\tilde{k}|^2 R^2 j_{\ell+1}(\tilde{k}R) j^*_{\ell+2}(\tilde{k}R) - (2\ell+3)\tilde{k}R j_{\ell+1}(\tilde{k}R) j^*_{\ell+1}(\tilde{k}R) + \ell(\ell+2) j_\ell(\tilde{k}R) j^*_{\ell+1}(\tilde{k}R)\big) + \text{c.c.}],\end{aligned} \tag{24}$$

as shown in Figs. 1 and 2 for fixed particle mass and object radius, respectively. Note, however, that this result is only valid for $\frac{n}{f} > k^2$; otherwise we are no longer in the scattering regime, so it is unclear whether the solution is still correct. Notice that the solution approaches the hard sphere limit as the potential increases, which makes sense because the hard sphere limit is precisely the limit of infinite potential. The solution also approaches the hard sphere limit as the radius increases, as shown in Fig. 3. This is because the hard sphere limit can also be thought of as the limit where the skin depth is significantly less than the radius of the object.

3 Phenomenology

Having calculated the observable, we can now look for experiments that might be sensitive to a new range of particle masses and interaction strengths that have not yet been excluded. The current exclusion limits for our particle are shown in Fig. 4 [4]. The region to the left of the diagonal line is the scattering region (the potential inside the scattering object is greater than the energy of the dark matter), so our calculations are only valid in this region. The horizontal lines indicate the radius required to be able to use the hard sphere approximation; the smaller the potential, the larger the scattering object must be. We considered

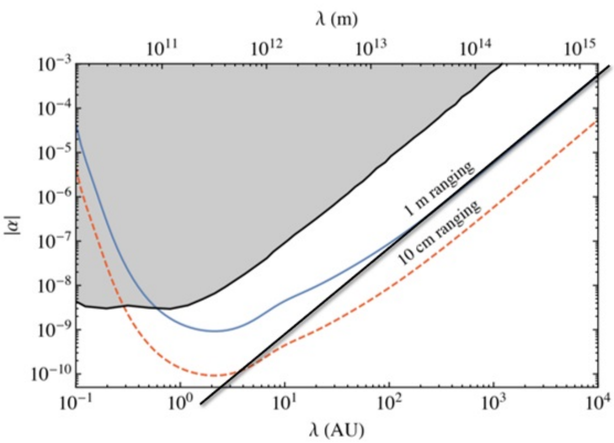


Figure 5: Sensitivity plot for the proposed experiment. The shaded region indicates the excluded values for the Yukawa potential. The blue solid line indicates the conservative sensitivity of the experiment, while the red dashed line indicates the optimistic sensitivity. The straight black line is added to emphasize the linear region of the sensitivity.

several experiments that might be sensitive to the region between the diagonal line and the excluded region. We found that earth-based experiments had too much shielding from the atmosphere and buildings to be sensitive, but several satellite experiments could potentially measure an effect.

The most promising experiment is one that was proposed to measure deviations from Newtonian gravity due to a new Yukawa force [5]. The experiment would send a satellite out perpendicular to the solar system, and at 1 AU (astronomical unit) would begin measuring accelerations of a test mass. This test mass would be surrounded by a large shield to prevent it from being affected by space debris and other unwanted external forces. The Yukawa correction to the gravitational potential is given by

$$\Phi(D) = -\frac{GM_\odot}{D}(1 + \alpha e^{-D/\lambda}), \tag{25}$$

where D is the distance from the solar system, $M_\odot$ is the mass of the sun, and α and λ are the unknown parameters of the theory that the experiment is trying to bound. For large λ, the leading order observable correction to the force is

$$\delta F = \frac{\alpha G M_\odot m}{2\lambda^2}, \tag{26}$$

where m is the mass of the satellite test mass. The sensitivity plot for the experiment is shown in Fig. 5. Using the linear region of the estimated sensitivity for large λ, and knowing that they use a 10 kg test mass, we find that the experiment is sensitive to forces as small as 10^{-13} N $= 10^{-19}$ GeV^2. We then compared this force to the force we would expect from the dark matter, given that the radius of the experiment's test mass is 5 cm, and we found that the experiment should be sensitive to the new range of masses and interaction strengths shown in Fig. 4.

4 Conclusion and Plans for Future Study

We calculated the force that dark matter would exert on ordinary matter if dark matter is composed of a particular light dark matter particle, and we found that a proposed experiment [5] could be sensitive to a new range of masses and interaction strengths that have not yet been excluded for our light dark matter particle. We are still working to determine whether our calculations are valid when we are no longer in the scattering regime, and we are also working to determine the ideal experimental configuration that is sensitive to the largest range of masses and interaction strengths that have not yet been excluded. This includes considering other observables that might allow us to be sensitive to a different region of the exclusion plot.

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