Designer H(z): Toward Exploring a Cosmological Solution to the Hubble Tension

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Abstract

Direct astrophysical observations find a rate of expansion of the universe of $H_0 = 73.52 \pm 1.62$ km/s/Mpc while indirect methods using the Cosmic Microwave Background and the standard cosmological model to map to today find $H_0 = 67.27 \pm 0.60$ km/s/Mpc. These values differ by 4.4σ and are a major area of debate in modern cosmology. We aim to explore a cosmological solution to this famous "Hubble Tension" by including a generalized energy component or scalar field into the Boltzmann code and doubling the model parameter space to include the rate of expansion over six epoch bins of redshift. This will allow the data to guide the model instead of forcing a specific model to the data. This preliminary work addresses the use of MCMC for cosmological posterior determination and the use of a specific parameter estimation architecture for cosmological fits. We find that our architecture reproduces Planck results for five out of six of the parameters, and include potential further study.

Background

Expansion of the Universe

The universe is expanding, and has been throughout cosmological history. The nature of this expansion constrains theories about the exact composition of energy in the universe. Today, for example, the universe is expanding at an accelerated rate due to the dominance of dark energy. This expansion is not the motion of objects through space, but rather an expansion of the space itself. In a given time interval when the universe has doubled in size, the space between nearby galaxies has doubled and the space between distant galaxies has doubled. This means that, from the perspective of the Earth, distant galaxies will appear to move away faster than closer galaxies.

It was Hubble who discovered this disparity in galactic motion in 1929 by comparing the recessional velocity derived from cosmological redshift measurements in spectra to the distance to the galaxy from which the spectra were taken. Hubble found that for nearby galaxies,

$$v = H_0 d,$$

where H_0 is a constant called the Hubble constant, v is recessional velocity, and d is distance. H_0, then, gives the recessional velocity of nearby objects per length away and is thus a measure of the rate of expansion today. Hubble's original measurements gave $H_0 = 100$ km/s/Mpc, meaning that galaxies separated by 1Mpc would have relative velocity 100km/s today.

The Hubble Tension

Hubble's method of finding the rate of expansion was a direct observational one, akin to measuring the velocity of cars on the road with radar. One could also imagine an indirect method akin to predicting the velocity of a car on one road based on its velocity on a previous road in its path and a model for its motion over time. These methods are completely independent, and if the velocity measurements on both roads and the model are correct, then these methods should agree. Indeed there are observations from the universe in the past: the Cosmic Microwave Background, and there is a model for changes in the universe over time. However, the values of the Hubble constant found by these two methods disagree by 4.4σ.

The direct observational method has been honed over the last 90 years, including, in addition to better telescopes and digital processing, the ability to measure distances of further galaxies. Cosmological distances are found using the distance ladder: a series of methods of measuring distances to further and further objects where each method is calibrated by data from the previous method. The distances to nearby objects ($d < 25$ million light years), namely Cepheid variable stars, are found using parallax. At different times throughout the year an object appears to be shifted based on the differential position of the Earth in its orbit and according to the distance to the object, similar to looking at an object with one eye closed and then switching eyes. This method is limited since the shift is smaller and more difficult to resolve for farther objects.

One type of prevalent nearby object, namely Cepheid variable stars, have an empirical relation between their period and luminosity. Parallax measurements for a population of nearby Cepheids allow this relation to be calibrated to distance and the stars to be used as so-called "Standard Candles" where the relation is extended to find the distances to farther galaxies containing Cepheid variable stars. The third and final rung of the distance ladder takes a similar step using type 1a supernovae. Distances to nearby (d < 100 million light years) supernovae are calibrated using the previous rung of the distance ladder for galaxies containing both Cepheid variable stars and type 1a supernovae. These supernovae are then assumed to be Standard Candles due to the universal thermonuclear process causing them, and the calibrated relation is extended to determine distances to more distant galaxies (d < 1 billion light years).

The current value from standard candles using HST (the Hubble Space Telescope), Gaia, and SH0ES (Supernovae, H0, Equation of State of Dark Energy) distance and redshift measurements is $H_0 = 73.52 \pm 1.62$ km/s/Mpc (Reiss et al, 2018). One group of proposed solutions to the Hubble tension claims that systematics in the distance ladder are artificially inflating the value. For example, the luminosity of supernova depends on the age of the stars in the local environment and galaxies at higher redshifts have higher star formation rates with more new, young stars. This could introduce a systematic bias in the distance ladder that would need to be understood and accounted.

Indirect determination of the Hubble constant relies on data from the Cosmic Microwave Background (CMB) along with a cosmological model. The CMB is the light emitted immediately after recombination, when electrons combined with protons into atoms for the first time. This is the time at which the universe became transparent and thus the CMB gives a snapshot of matter perturbations in the universe at that time (380,000 years after the big bang). CMB data include temperature (T) as well as E- and B- mode polarization. Power spectra can be constructed from correlations of modes as a measure of the amplitude of fluctuations on different angular scales. For example, the TT spectrum contains information about temperature fluctuations, while the TE spectrum gives the cross-correlation between temperature and E-mode polarization. Fitting these data with a model that depends on H_0 provides an indirect value of the Hubble constant.

The standard cosmological model, called ΛCDM, includes the cosmological constant, or dark energy, and cold dark matter. This model includes six parameters: the energy density of dark matter, Ω_{CDM}; the energy density of baryons, Ω_b; the angular sound horizon, θ_s; the optical depth, τ; an amplitude factor, A; and the spectral density, n_s. The θ_s parameter is degenerate with H_0 and is what is known as a "Standard Ruler". The angular sound horizon is related trigonometrically to the distance to the CMB (d_A) and the physical radius of the sound horizon at the time of the CMB (R_{phys}) by,

$$\theta_{obs} = \frac{\text{comoving length of ruler}}{\text{comoving distance to ruler}} = \frac{R_{phys}/a_{ruler}}{d_A/a_{ruler}} = \frac{\int_0^{a_{CMB}} \frac{c_s\,da}{a^2 H_1(a)}}{\int_{a_{CMB}}^1 \frac{da}{a^2 H_2(a)}}$$

where H_1 is the expansion rate before the time of the CMB and depends only on the densities of radiation and matter, which can be found from the CMB temperature and parameter estimates, respectively; while H_2 is the expansion rate after the CMB , which scales back from today using the relative energy density of different components of the universe, and thus depends on H_0. In this way, the Hubble constant can be either derived from parameterization of the sound horizon or explicitly included as a parameter in the CMB power spectrum fit. (Dodelson, 2003).

The current value using this method and assuming the standard cosmological model is $H_0 = 67.27 \pm 0.60$ km/s/Mpc (Planck Collaboration VI, 2018). The second camp of solutions to the Hubble tension involve changes to the standard cosmological model and variations on power spectra fitting techniques.

Previous studies of cosmological solutions found that late-time solutions are inconsistent with constraints from other observations, such as baryon acoustic oscillations (BAO; Aylor et al, 2018), so that any changes to the cosmological model that would affect H_0 need to be in the two decades of expansion before recombination. The aim of the Designer H(z) project is to add a generalized new species or scalar field into the model to get more freedom for the expansion rate around the time of recombination, and to use a larger parameter space that includes multiple redshift epoch bins for H_0.

Methods

Cosmological models contain six or more parameters. This large parameter space makes sampling on a grid to find a fit computationally impractical. Additionally, it is useful to have full probability distributions for parameters rather than just single fit values with uncertainties. These considerations motivate the use of Markov Chain Monte Carlo.

Markov Chain Monte Carlo

Markov chain Monte Carlo methods provide a means of sampling probability functions computationally using a chain of samples analyzed against a prior and likelihood. According to Bayes' Theorem in statistics,

$$P(\theta|D) \propto P(\theta)P(D|\theta),$$

where P(x) is the probability of x, θ is an ordered set of parameters, and D is observed data. $P(\theta|D)$ can be interpreted as the posterior, that is, the probability distributions of the parameters given the observed data; $P(\theta)$ as the prior, the probability distribution of the parameters independently; and $P(D|\theta)$ as the likelihood, the probability that the observed data occurred given a set of parameters. This is shown graphically for a one-dimensional parameter space in figure 1. This means that, having observed some data, and knowing the dependence of the data on the parameters, we can find the distribution of parameters conditional on the observed data. In a multi-dimensional parameter space where the independent values of parameters are of interest, marginal probabilities can be found by integrating the probability of the full parameter vector given the data over the reduced space of all parameters except the one of interest,

$$P(\theta_i|D) = \int P(\theta_1, \theta_2, \ldots, \theta_n|D) d\theta_1 d\theta_2 \ldots d\theta_{i-1} d\theta_{i+1} \ldots d\theta_n$$ (Gilks et al, 1996).

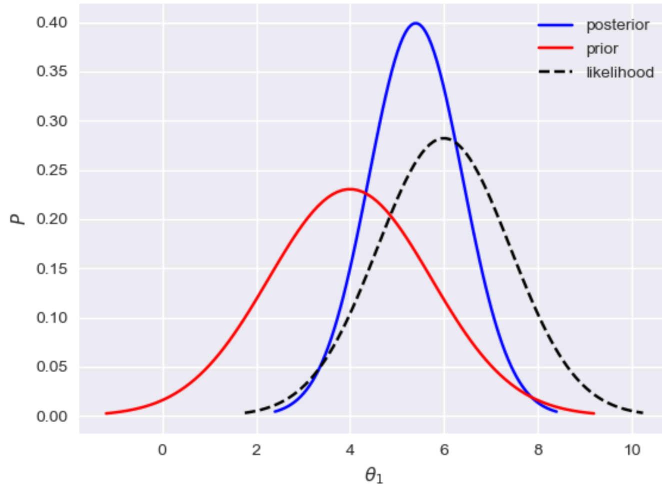

Figure 1. Graphical representation of Bayes' theorem with Gaussian probabilities

We used the Metropolis Hastings algorithm to implement our chain. This algorithm samples a value of the parameter vector based on the current value in the chain and the prior distribution. The algorithm will either accept or reject the new sampled value based on its likelihood. If it is accepted then the sample value will be kept as the value at this step in the chain. If it is rejected then the chain will repeat the value it had before sampling (Hastings, 1970; figure 2a). After some burn-in steps, the chain begins to sample the posterior (figure 2b).

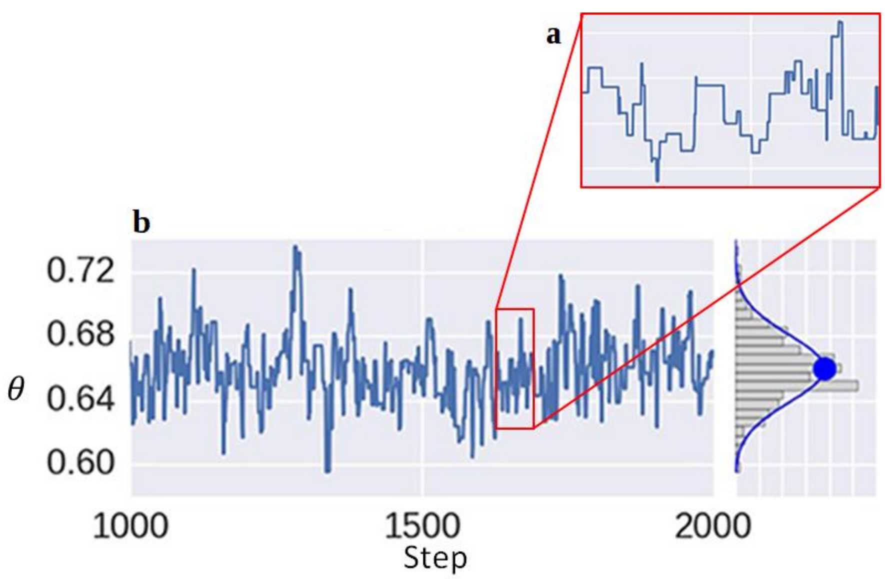

Figure 2. (a) example Metropolis Hastings chain for a single parameter and (b) zoomed-in view to show jumps to a new value at acceptances and constant value at rejections

Cosmological posterior determination

Before we can expand the parameter space to include bins of epochs of expansion, we needed to check that we can use our posterior determination code architecture to reproduce the Planck posteriors. Our full architecture began with priors, which include constraints from other cosmological observations such as BAO and which we used from Planck Collaboration VI (2018). We run a Metropolis Hastings algorithm on these priors and allow it to accept and reject samples based on the likelihood. We found the likelihood using a comparison between the CMB data and output from a Boltzmann solver. The Boltzmann solver, CLASS (Cosmic Linear Anisotropy Solving System; Blas et al, 2011), numerically evolves density and velocity perturbation equations for each species of energy in the universe based on the sampled model parameters and outputs a modeled CMB power spectrum. As an example, figure 3 shows CLASS model power spectra for three different values of H_0.

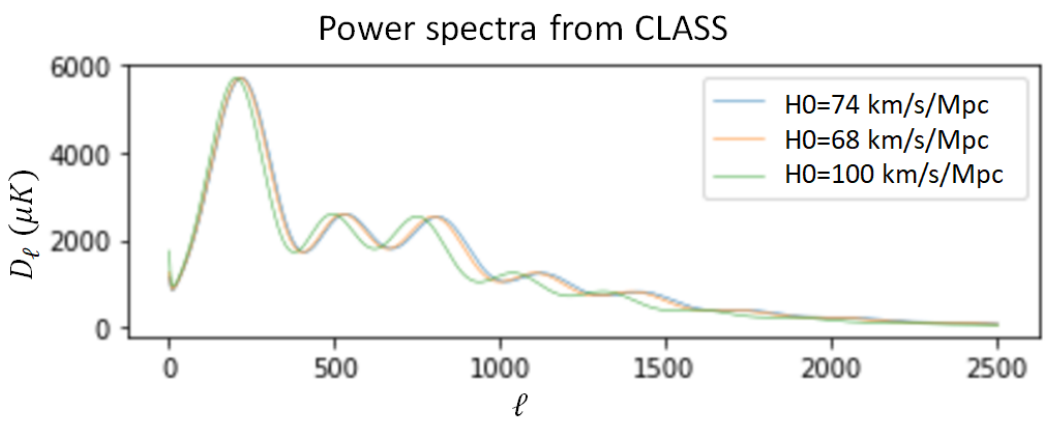

Figure 3. Example CLASS power spectra for different values of the Hubble constant. (ℓ gives the angular mode, where smaller modes correspond to larger perturbation wavelength scales; Blas et al, 2011)

Results

We are able to use our cosmological parameter posterior determination architecture for the six-parameter standard cosmological model. We are able to reproduce the Planck posteriors for all parameters except θ_s (figure 4 and Appendix). Though the uncertainty on θ_s is very small, O(-4), the posteriors differ by about 1σ.

One explanation for such an offset in MCMC posteriors is a lack of convergence. We ran the chains for 20,000 steps and the acceptance ratio was R=0.23 which is well within the ideal 0.2-0.3 range. We also used the covariance matrix generated from a full 20,000 step chain as the prior for a new 20,000 step chain and found indistinguishable curves, with only variations on the third significant digit of some standard deviations. We conclude that the chain is converging, albeit, to a different value from the Planck posterior.

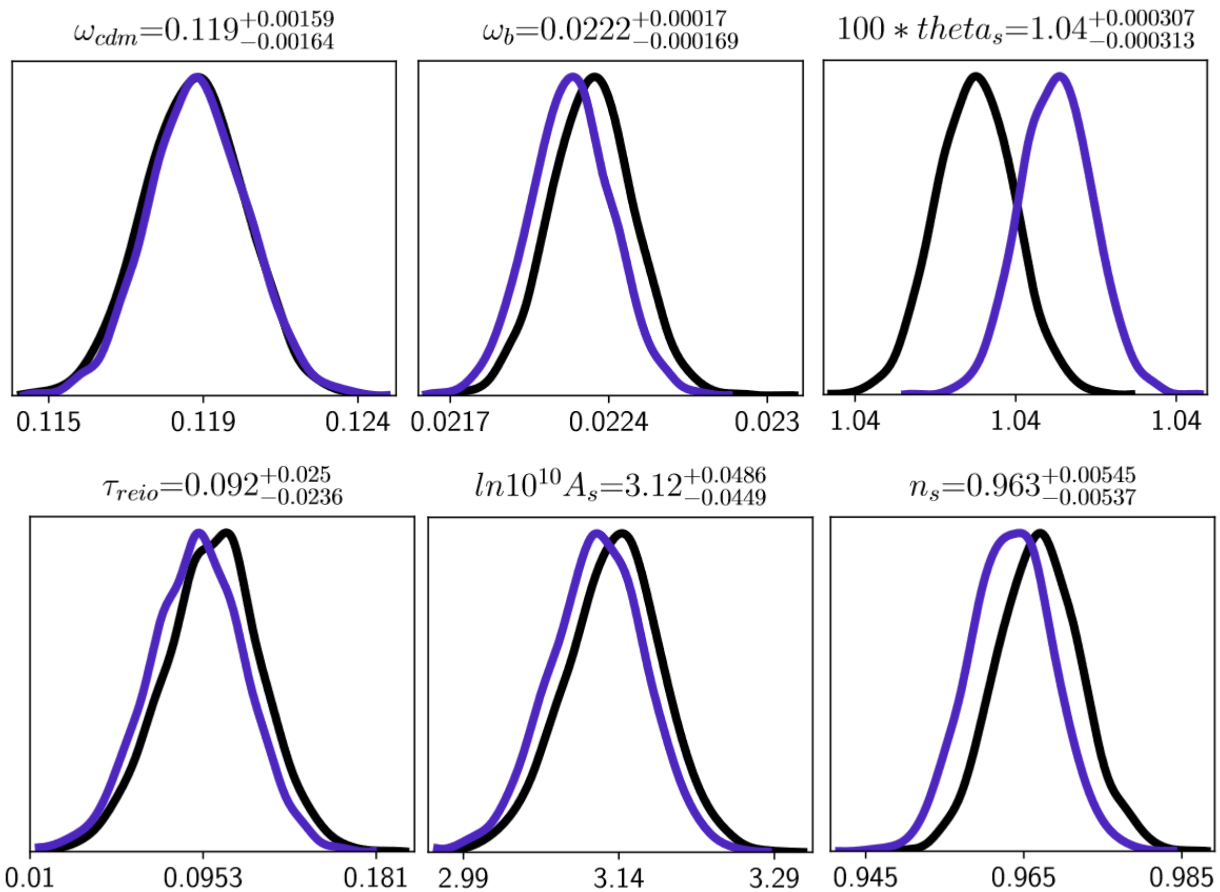

Figure 4. Individual marginal posterior distributions for 6-parameter standard cosmological model (black) compared to Planck Collaboration VI (2018) posteriors (blue)

Discussion

Preliminary use of our posterior determination architecture demonstrates that we can reproduce the Planck results, but have one parameter (θ_s) left to optimize. It is essential to optimize this parameter before moving on to higher-dimensional analysis due to its degeneracy with H_0.

Once the θ_s posterior is improved, we can use these methods to find posteriors on a 12-dimensional space with 6 new parameters being parameterization of H_0 in bins of redshift. By also adding in a new, flexible species that can dominate around the time of recombination, we can allow the data to design the model rather than forcing a particular model to the data. Based on previous work in this era before recombination we expect this kind of re-parameterization will widen the H_0 posterior and thus weaken the Hubble tension.

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Appendix

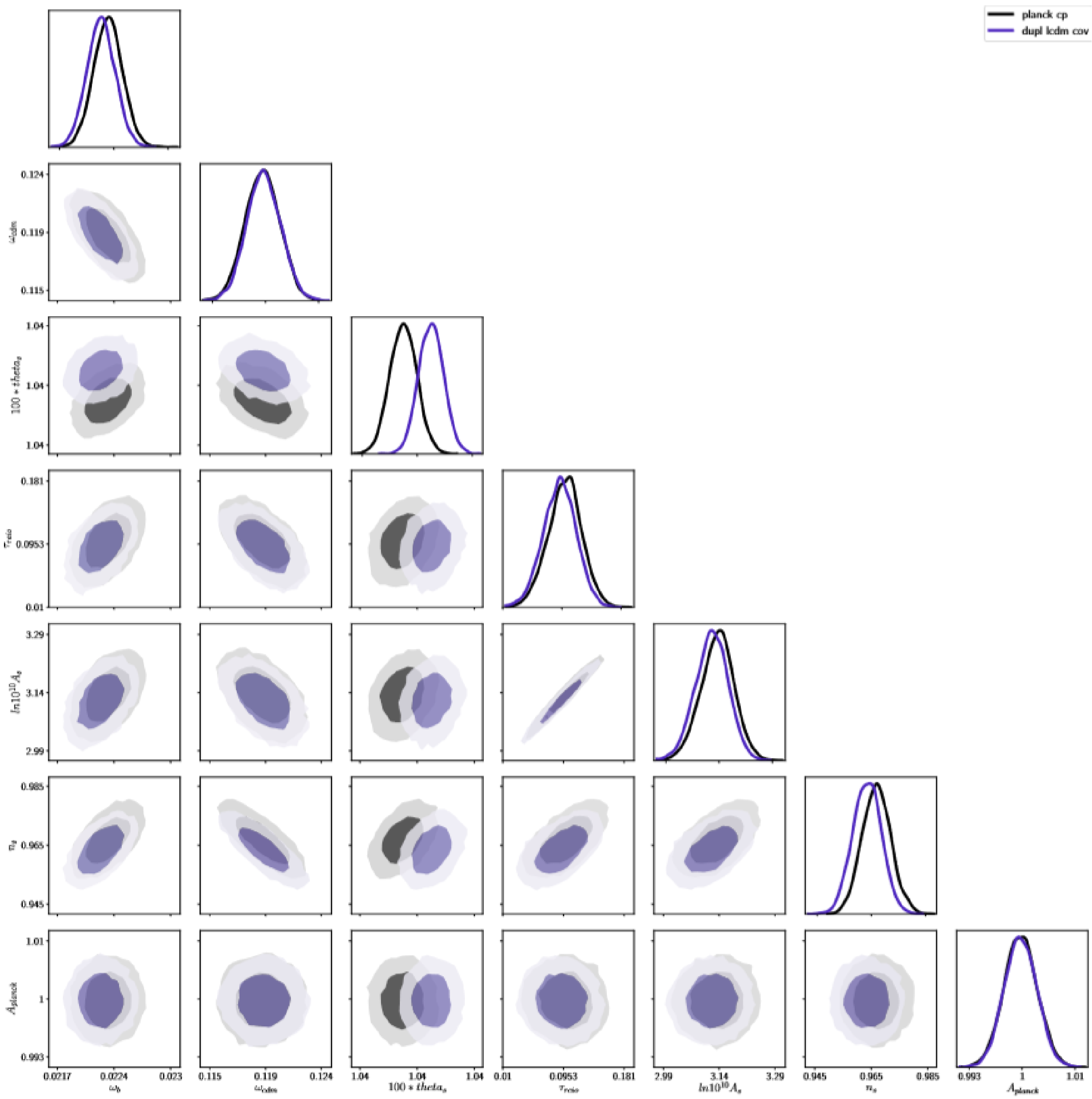

Figure A: Full marginal posterior distributions for 6-parameter standard cosmological model (black) compared to Planck Collaboration VI (2018) posteriors (blue)