Problem Set 7
Physics 240B

Due Thursday, March 12, 2009
No late homework accepted

1. a) Assume that a one-dimensional crystal with lattice constant a has exactly the monatomic chain dispersion relation. For a neutron incident on the crystal (with wave vector parallel to the crystal), show how a graphical solution for phonon *emission* by the neutron would look.

b) What minimum energy must the neutron have for phonon emission?

c) Describe qualitatively how the number of solutions for phonon emission depends on the incident neutron's energy.

2. (If you refer to Marder, be aware that, in the original printing, his neutron scattering discussion has typos and at least one significant misstatement.)

a) Assume a Debye model in three dimensions, with one pure longitudinal and two pure transverse modes, all having the same sound velocity. Show that the Debye-Waller factor at $T = 0$ is $2W = \frac{3}{4}\frac{q^2\hbar}{M\omega_D}$. The zero-temperature assumption means that you are only evaluating zero-point motion terms.

b) Show that at non-zero temperature, the Debye-Waller factor diverges in two dimensions but not in three dimensions.

3. Consider a cube of material with equal density n of charges $\pm e$. Imagine displacing the negative charges by $d\hat{\mathbf{x}}$ with respect to the positive charges. Show that the system sustains position oscillations of the charges at the plasma frequency.

4. Ionic crystals can be piezo-electric, and create an electric field proportional to *displacements* of the ions. The deformation potential interaction, in which the electric field is instead proportional to the spatial derivative of the atomic displacements, has matrix elements with $|g_{\mathbf{k}\mathbf{k}'}|^2 \propto |\mathbf{k} - \mathbf{k}'| = q$ for electron-phonon scattering.

a) From the above information and general Fourier space considerations, what do you expect for the q-dependence of the piezo-electric matrix elements?

b) If the piezo-electric interaction dominates electron-phonon scattering, what should be the temperature dependence of resistivity at $T >> \Theta_D$ and $T << \Theta_D$?

c) The deformation potential interaction is always present, even in a piezo-electric crystal. With both mechanisms in place, how does resistivity behave? In what regimes does it have a simple power-law form?