Problem Set 5
Physics 240B

Due Thursday February 19, 2009
Late HW accepted until class on Tuesday, February 24

Do A&M 22.2 and 22.3, plus the following:

1. (similar to A&M 22.1, but simpler)
We made the approximation that only nearest-neighbor interactions matter. To see how similar everything looks with further-range interactions added, consider a one-dimensional chain of identical atoms, equally spaced at equilibrium. Assume there are both nearest-neighbor interactions, with spring constant K_1, and next-nearest-neighbor interactions, with spring constant K_2. Find the equations of motion and dispersion relation. Assume $\mathrm{K}_2/\mathrm{K}_1$ is small, and expand $\omega(k)$ to first order in $\mathrm{K}_2/\mathrm{K}_1$. Finally, on a single graph, sketch $\omega(k)$ for both $\mathrm{K}_2 = 0$ and $\mathrm{K}_2/\mathrm{K}_1 = 0.1$.

2. Consider a 2-dimensional crystal with a square lattice structure. Model it as a system of masses and springs, with the springs connecting nearest neighbor atoms (spring constant K_1) and next nearest neighbors (spring constant K_2).

a) Write the equations of motion for a mass at position (ma, na). Keep only first-order terms in the displacements.

b) If $K_1 = K_2$, find the frequencies of the modes with $\mathbf{k} = \frac{k}{\sqrt{2}}(1, 1)$.

c) Repeat b) for $K_1 = 2K_2$.

d) Show that for $K_1 = 2K_2$, polarization in the x direction is impossible. (That is, $\epsilon_{\mathbf{k}\nu} \neq \hat{\mathbf{x}}$.)