Final Exam, Physics 240A

December 8, 2008

Do all questions; one problem is on the back. Explain all answers. Note that frequently you can do later parts of questions even if you didn't solve earlier parts.

Some possibly useful constants and conversions:

$k_B = 1.38 \times 10^{-16}$ erg/K	1.6×10^{-19} Coulomb=4.8×10^{-10} esu (electron charge)
$\hbar = 1.05 \times 10^{-27}$ erg-s	1 eV = 1.6×10^{-12} erg
$m = 9.1 \times 10^{-28}$ g (electron rest mass)	1 Tesla = 10^4 gauss

1. (10 points)
Explain whether you agree or disagree with the following statement, and why:
"The electron collisions that determine τ could be collisions with lattice ions in the Sommerfeld model, but not in the semiclassical model."

2. (20 points)
A one-dimensional Bravais lattice with lattice constant a has a dispersion relation $\varepsilon(k) = 1 - \cos ak + \sin^2 ak$ in the first Brillouin zone. An electric field is applied in the positive direction.

a) In the semiclassical model, explain how electrons that begin in the states at $k = \pi/6a$, $k = \pi/2a$, and $k = \pi/a$ develop over time. You should give each electron's initial velocity, how k changes with the field, and whether each electron accelerates or decelerates due to the field. Also note which results are different from the classical expectation.

b) Find the effective mass of electrons at each maximum and minimum in the band.

3. (40 points)
Consider a two-dimensional rectangular lattice with primitive vectors $\mathbf{a_1} = a\hat{\mathbf{x}}$ and $\mathbf{a_2} = 2a\hat{\mathbf{y}}$, where $a = 2.3$ Å.

a) What is the reciprocal lattice?

b) Sketch the first three Brillouin zones of the reciprocal lattice.

c) Find the real-space conduction electron density for which the free electron Fermi surface first touches the edge of the first Brillouin zone. (You need to derive the formula relating the Fermi level and the electron density.)

d) If the real-space conduction electron density is 3.6×10^{15} per cm^2, what does the free electron Fermi surface look like in each of the first three Brillouin zones? Map your answers back to the first zone.

e) How would you expect a small lattice potential U to affect the Fermi surface pieces from part d)? (Re-draw the Fermi surface and indicate the changes.)

f) The state derived from the free-electron wave function with $\mathbf{k} = (\frac{3\pi}{2a}, \frac{\pi}{2a})$ couples most strongly to states derived from which other free-electron wave functions? In each case give the wave vector $\mathbf{q}$ of the relevant Fourier coefficient $U_{\mathbf{q}}$.

g) If none of the Fourier coefficients of the potential vanish, what additional Fourier coefficients, beyond those listed in f), must be considered when calculating the first-order correction to the free-electron energy at $(\frac{3\pi}{2a}, \frac{\pi}{2a})$? Why?

4. (30 points)
The de Haas-van Alphen effect involves oscillations in the magnetic susceptibility χ of a metal in a high magnetic field.

a) What important piece of information about a crystal does the de Haas-van Alphen effect measure?

b) Susceptibility measures the induced magnetic moment, $\chi = dM/dH$. In this case the moment comes from an alignment of the magnetic moments of the conduction electrons caused by a term in the Hamiltonian proportional to $\mathbf{S} \cdot \mathbf{H}$. Consider a metal at *low* magnetic fields, where Landau levels can be neglected in favor of an energy band picture, and assume the applied field changes only the occupation probabilities of the orbital states, not the states themselves. At low but finite temperature, sketch the occupation probability as a function of the orbital energy for:

i) zero applied magnetic field

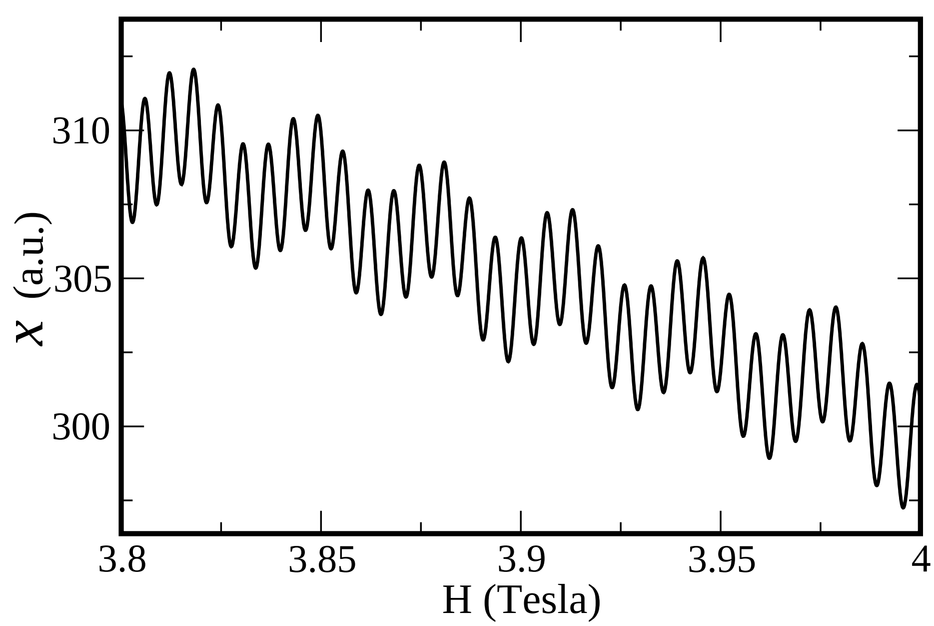
ii) small field $\mathbf{H}$, for states with spin up only

iii) small field $\mathbf{H}$, for all states

Use your graphs to explain why $\mathbf{M}$ changes with $\mathbf{H}$.

c) For large $\mathbf{H}$, the minima in χ occur at either complete filling or half-filling of the highest Landau level. Which one, and why? Similarly, where do the maxima occur, and why?

d) The Fermi surface of a certain material has ellipsoidal pieces with two major axes equal in length and the third a factor of five longer. Not all the ellipsoids have the same orientation. For a magnetic field applied parallel to the long axis of some ellipsoids and perpendicular to the long axis of other ellipsoids, the magnetic susceptibility behaves as shown below. What are the lengths of the ellipsoid axes?



e) What limit can you put on temperature for the data in part d)?