

Answer Set 1

Physics 240B

A&M 28.2 a) The field is at one angle with respect to the two pockets along the z axis, and at another angle with respect to the four remaining pockets. The two angles give rise to two effective masses, and hence two field magnitudes for resonance.

b) The components of $\hat{\mathbf{H}}$ are $H_3 = \cos 30 = \sqrt{3}/2$, $H_1 = H_2 = \sin 30 \cos 45 = 1/\sqrt{8}$. For the pockets along the z axis, $m_1 = m_2 = 0.2m$ and $m_3 = m$. Then from equation (28.8), $m^* = \sqrt{0.04m^3 / (\frac{1}{8}(0.2) + \frac{1}{8}(0.2) + \frac{3}{4})m} = 0.22m$. The field for resonance is $H = m^*c\omega/e = (0.22)(9.1 \times 10^{-28})(3 \times 10^{10})(2\pi)(2.4 \times 10^{10})/4.8 \times 10^{-10} = 1887$ gauss, so this is the lower peak in the figure. For the remaining four pockets, m_3 and one of the other two equal $0.2m$, while the remaining mass eigenvalue is m . The cyclotron effective mass becomes $m^* = \sqrt{0.04m^3 / (\frac{1}{8}(0.2) + \frac{1}{8} + \frac{3}{4}(0.2))m} = 0.37m$, and $H = 3173$ gauss, the upper peak.

c) First of all, note that the vector (1,1,1) does *not* make a 45° angle with the xy plane. In fact, the geometry is too hard for me to visualize, so I'll put in coordinates and crank. Unit vectors in the directions of the four pockets are $(\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}})$. By dotting the unit vector along the magnetic field, $(\frac{\sqrt{3}}{2\sqrt{2}}, -\frac{\sqrt{3}}{2\sqrt{2}}, \frac{1}{2})$, with each of these, we get the angle of the field and the long axis of the pocket. I get $\cos \theta = 0.29$ (twice), 0.42 , and 0.996 . Since there are three distinct values, there are three cyclotron resonance peaks.

d) Since the two transverse masses are equal, all that matters is $H_1^2 + H_2^2 = \sin^2 \theta$. For each of the angles in c), we get $m^* = \sqrt{1.6(.08)(.08) / (.08 \sin^2 \theta + 1.6 \cos^2 \theta)}$. The $[1\bar{1}\bar{1}]$ pocket gives $m^* = \sqrt{1.6(.08)(.08) / (.008(.08) + .992(1.6))m} = .080m$, and $H = 690$ gauss (lowest peak). For the $[1\bar{1}\bar{1}]$ pocket, $m^* = \sqrt{1.6(.08)(.08) / (.824(.08) + .176(1.6))m} = .17m$ and $H = 1500$ gauss (middle peak). For the $[111]$ and $[1\bar{1}\bar{1}]$ pockets, $m^* = \sqrt{1.6(.08)(.08) / (.916(.08) + .084(1.6))m} = .22m$ and $H = 1900$ gauss (upper peak).

A&M 28.6 a) Some of the electrons available because of the donor states drop down and fill the acceptor states (which are at much lower energy). Hence $p_a \ll N_a$. The relevant equation comes from the occupation probability of each acceptor state: $p_a = N_a(1 - \frac{2e^{(\mu - \varepsilon_a)/kT}}{1 + 2e^{(\mu - \varepsilon_a)/kT}})$. (The fraction is subtracted from one because the hole density equals the density of *unoccupied* electron states. Also, I'm using ε_a as the energy of the acceptor states, rather than the energy difference between the acceptors and the top of the valence band as I did in class.) Since $N_d > N_a$, the chemical potential is higher than the middle of the gap, so $\mu - \varepsilon_a$ is large compared to kT and $p_a/N_a \ll 1$.

If N_d is large enough that μ moves up significantly, then the vast majority of electrons in the conduction band must come from donor states rather than from the valence band and $n_c \gg p_v$.

Finally, if (28.10) held, so that essentially all the donors were ionized, then the conduction electron density would be $n_c = N_d - N_a$. (This is what's left after some donor electrons drop down to fill the acceptor states.) However, at low enough temperatures, the density of non-ionized donors may be significant. This gives (28.45), $n_c = N_c e^{-(\varepsilon_c - \mu)/kT} = N_d - N_a - \frac{N_d}{\frac{1}{2}e^{(\varepsilon_d - \mu)/kT} + 1}$.

b) Equation (28.45) gives two equations for the unknowns n_c and μ . Solving both for μ gives

$\mu = \varepsilon_c + kT \ln \frac{n_c}{N_c} = \varepsilon_d - kT \ln \frac{2(N_a + n_c)}{N_d - N_a - n_c}$. From this, solve for n_c in terms of known quantities only (i.e., no μ): $\frac{2n_c(N_a + n_c)}{N_c(N_d - N_a - n_c)} = e^{-(\varepsilon_c - \varepsilon_d)/kT}$. At low enough temperatures that $N_a \ll n_c \ll N_d - N_a$, this becomes $n_c^2 = \frac{N_c(N_d - N_a)}{2} e^{-(\varepsilon_c - \varepsilon_d)/kT}$, which immediately gives (28.46).

- c) As n_c falls even more, eventually $n_c \ll N_a, N_d - N_a$. The general equation for n_c in part b) now becomes $n_c = \frac{N_c(N_d - N_a)}{2N_a} e^{-(\varepsilon_c - \varepsilon_d)/kT}$.
1. For electron energies $\varepsilon(\mathbf{k}) = \varepsilon_c + \hbar^2 \left(\frac{k_1^2}{2m_1} + \frac{k_2^2}{2m_2} + \frac{k_3^2}{2m_3} \right)$, where \mathbf{k} is a vector from the band minimum rather than from the usual k -space origin. The constant-energy surface at ε has volume $\frac{4\pi}{3} \frac{\sqrt{2m_1(\varepsilon - \varepsilon_c)}}{\hbar} \frac{\sqrt{2m_2(\varepsilon - \varepsilon_c)}}{\hbar} \frac{\sqrt{2m_3(\varepsilon - \varepsilon_c)}}{\hbar} = \frac{4\pi}{3\hbar^3} (2(\varepsilon - \varepsilon_c))^{3/2} \sqrt{m_1 m_2 m_3}$. Differentiating and dividing by $4\pi^3$ for the density of states in k -space, $g_c(\varepsilon) = \sqrt{2(\varepsilon - \varepsilon_c)} \frac{m_c^{3/2}}{\hbar^3 \pi^2}$. Next evaluate the integral for N_c : $N_c(T) = \sqrt{2} \frac{m_c^{3/2}}{\hbar^3 \pi^2} \int_{\varepsilon_c}^{\infty} d\varepsilon \sqrt{\varepsilon - \varepsilon_c} e^{-(\varepsilon - \varepsilon_c)/kT} = \sqrt{2} \frac{m_c^{3/2}}{\hbar^3 \pi^2} 2(kT)^{3/2} \int_0^{\infty} dx x^2 e^{-x^2}$, where $x = \sqrt{(\varepsilon - \varepsilon_c)/kT}$. Use integration by parts and the fact that $\int_0^{\infty} dx e^{-x^2} = \sqrt{\pi}$ to evaluate the integral; it works out to $\sqrt{\pi}/4$. This gives $N_c(T) = \frac{1}{4} \left(\frac{2m_c kT}{\pi \hbar^2} \right)^{3/2}$. Finally, take $k = 1.4 \times 10^{-16}$, $\hbar = 1.05 \times 10^{-27}$, and $m = 9.1 \times 10^{-28}$ (all cgs units). I actually get 2.6×10^{19} rather than 2.5 for the prefactor of (28.16).
2. a) The conduction band has larger effective mass, so it is shallower than the valence band.
- b) The conduction band density of states grows more quickly (i.e., goes out more horizontally at a given energy) than does the valence band, again because of the effective mass difference.
- c) Using the handy numerical formulas (28.16) and the law of mass action, $n_i = n_c = p_v$ is $33/\text{cm}^3$ at 300K and $10,000/\text{cm}^3$ at 350K. (The exact values change quite a bit depending on how many digits you keep in the Kelvin-eV conversion in the exponential.) Then use $\mu = \frac{\varepsilon_v + \varepsilon_c}{2} + \frac{3}{4} kT \ln \frac{m_p^*}{m_n^*}$ to get that μ is 11 meV (13 meV) below the gap center at 300K (350K).
- d) Even $10^{11}/\text{cm}^3$ is huge compared to the intrinsic carrier density, so $p_v = 10^{11}/\text{cm}^3$ at both temperatures and from law of mass action again $n_c \approx n_i^2/N_a = 1.2 \times 10^{-8}$ or 1.0×10^{-3} per cm^3 at the two temperatures. The chemical potential now has an extra term, $-kT \sinh^{-1} \frac{N_a}{2n_i}$, in addition to the motion caused by the effective mass difference. It is now -0.58 eV or -0.50 eV below the gap center.
- e) The doped semiconductor is more likely to have temperature-independent resistance, since the carrier density is nearly constant. (The minority carrier density changes substantially with temperature, but this doesn't much affect the material's conductivity.) There can also be temperature dependence from the carrier mobility, essentially the mean free path. If the collisions are dominated by impurities, this will also be nearly temperature-independent.