# Kelvin Wave Oscillations in Superfluid Helium Vortices

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**Abstract**

This work describes the behavior of Helium 4 vortices studied during the summer of 2018, along with the experimental setup used. New data has revealed higher harmonic Kelvin waves on vortex cores as well as provided further insight into continuous excitation measurement techniques. In addition, interesting vortex signatures were observed during pinning and when high frequency pulsed measurements were taken that must now be explored further. Finally, a new two-frequency experimental technique was explored. The data collected in these experiments must now be analyzed in order to draw quantitative conclusions.

# 1 Introduction

## 1.1 Vortices

Vortices, or regions of fluid flow around an axis, are relatively common occurrences in the macroscopic world. They can be observed during storms in the form of tornadoes, around the wings of an airplane, or even in the air around a whistling mouth [1]. Understanding these phenomena can be crucial to developing improved technologies, such as safer airplanes, or to understanding weather patterns more thoroughly. Thus, the study of vortices is an important field that has the potential to greatly expand and improve vortex applications.

## 1.2 Superfluids

In normal fluids, vortices can quickly become complicated when they interact with other vortices or change over time. For this reason, it can be beneficial to study them in simplified systems, such as superfluid helium. There are two kinds of superfluid helium: He-4 and He-3. When He-4 reaches a temperature of approximately 2.17 Kelvin, the individual particles all occupy their lowest energy state and the fluid condenses into a superfluid. In He-3 the particles act as fermions, so they must pair up before they can condense into the same state. Due to the weak pairing interaction, this condensation happens around 1 mK in He-3 [2]. Just as superconductors have no resistivity, superfluids have no viscosity. In addition, they have quantized circulation, which can be described by,

$$\kappa = \oint v dl = \frac{hn}{m_{He}} \tag{1}$$

where $\kappa$ is the circulation, $v$ is the fluid velocity, $h$ is Planck's constant, $n$ is the quantum number, and $m_{He}$ is the mass of a helium atom [2]. Thus, when a vortex is created in a superfluid, the velocity of the fluid decays with $\frac{1}{r}$ and the curl is zero. In order for the curl to be zero everywhere, vortices in superfluid helium must have a normal core, which is exploited in these experiments. This clear vortex core, which has a size of about 3 $\mathring{A}$, in combination with the quantized circulation also means that the velocity field can be well-defined everywhere in the contained. Finally, the lack of viscosity means that vortices are relatively stable over time as they do not dissipate energy.

Though all superfluid helium exhibits the properties described above, when it has a temperature greater than 0 K it is best modeled as a combination of normal fluids and superfluids. As the temperature decreases below the transition temperature, the normal fluid contribution also decreases [3]. For this reason, in combination with the very low temperatures required to drive He-3 superfluid, these experiments were carried out with superfluid He-4 at temperatures around 300 mK.

# 2 Experimental Setup

## 2.1 The Fridge

In order to reach these very low temperatures, a He-3 fridge, shown in Fig. 1, was used to cool the He-4 in three steps. In the first step, the fridge is submerged in a dewar of liquid He-4, which is cheaper than He-3. The vacuum can surrounding the experimental cell is filled with an exchange gas, He-4, which creates a thermal link between the cell and the Helium in the dewar bringing the cell down to about 4.2 K [4].

In the next cooling step, He-4 is siphoned from the dewar and brought to the He-4 plate. A mechanical pump at the top of the fridge pumps on this plate, causing the He-4 to evaporate and reducing the temperature to about 1.5 K [4]. In the final step of the cooling process, the He-3 Charcoal pump reaches about 20 K and begins to absorb He-3 stored in the He-3 pot near the bottom of the fridge. The evaporation of the He-3 brings the temperature to the desired temperature of 300 mK. The exact temperature can the be controlled with heating elements at the charcoal pump.

Because He-3 is very expensive, there is a finite amount of the gas contained within the fridge. Thus, when all of the He-3 is absorbed into the charcoal pump and the He-3 pot is empty, no further cooling can occur and the fridge begins to warm. At this point, a restart is initiated by heating the charcoal pump so that it releases the He-3 back into the pot. When the experimental cell is filled with He-4, the hold-time of the fridge is approximately 3 hours, which severely limits how many measurements can be taken during a single cool-down. When the fridge warms up, the He-4 in the cell rises above 2.17 K and is thus no longer superfluid. Therefore, any vortices created when the fridge was cold are lost, and each vortex can only be studied for a limited amount of time.

## 2.2 The Vibrating Wire

In order to measure vortices in superfluid helium, a vibrating wire cell is used. This brass cell, shown in Fig. 2, is about 2-3 inches in length and has a thin superconducting wire stretched vertically through it. The wire is secured at each end of the cell with stycast caps. The top cap has a small hole through which He-4 can flow into the cell. The brass base at the top of the cell has a copper-nickel tube through which the superconducting wire is connected to the measurement wires on the fridge. This cell is then secured to the bottom of the fridge using an indium O-ring and screws. Two superconducting magnets are placed around the cell on the fridge to create an adjustable magnetic field perpendicular to the wire. Thus, the motion of the wire in the cell can be controlled by running a current through the it.

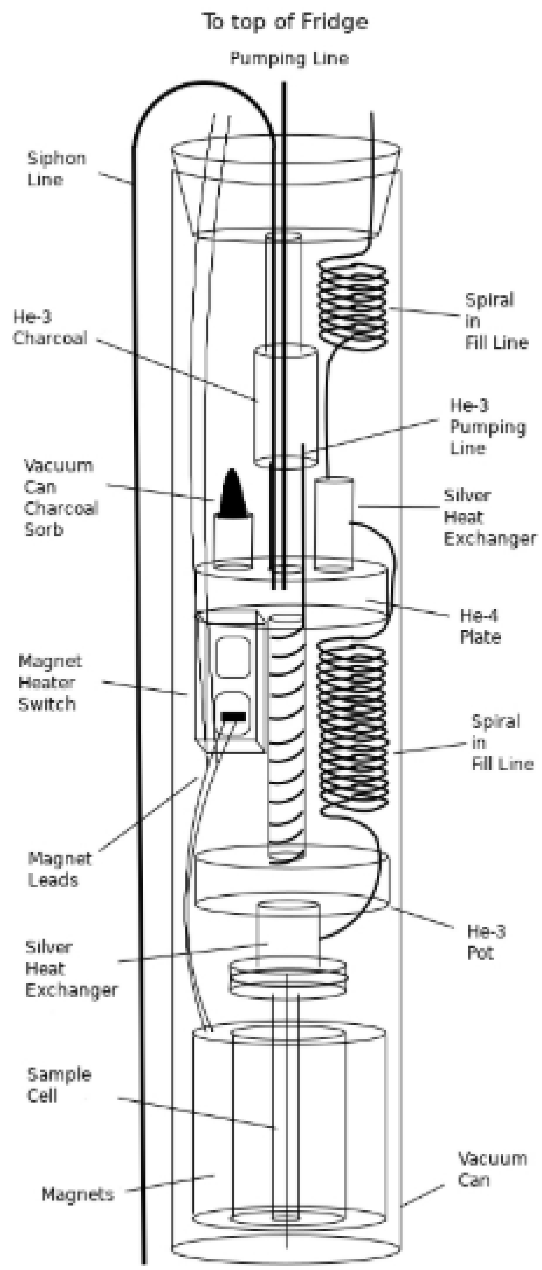


Figure 1: A diagram of the Helium-3 fridge that is placed into a vacuum can and then a dewar of He-4. The He-4 is siphoned in to the He-4 Plate through the siphon line on the left to reduce the temperature to about 1.5 K, while the charcoal pump for the He-3 near the top of the fridge pumps on the He-3 Pot to further cool the system. The experimental vibrating wire cell is mounted at the bottom of the fridge and surrounded by two superconducting magnets which create a magnetic field perpendicular to the wire.

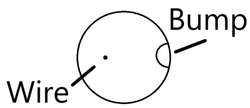


Figure 2: Left: An image of the vibrating wire cell before it is mounted on the fridge. The cell is about 3 inches long and 1.5 inches in diameter. The wire spans it vertically under tension. The bump is located about halfway down the side of the cell. Right: A schematic of a top-down view of the vibrating wire cell. The wire runs through the middle and the bump is attached to the cell wall.

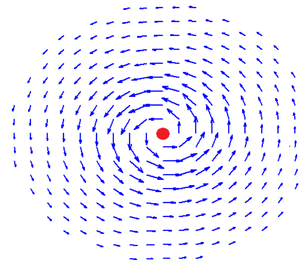

Figure 3: A diagram of a $1/r$ velocity field like the one that surrounds the vibrating wire. In this schematic, the wire, represented by the red circle, passes perpendicularly through the field into the page.

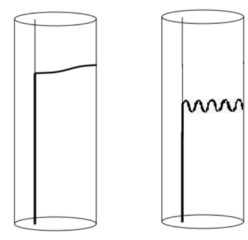

Figure 4: Left: A diagram of the vibrating wire cell when the vortex core detaches from the wire and extends instead to one of the cell walls. [2]. Right: A sketch of Kelvin wave vibrations on the free vortex core.

The vibrating wire cell can be used to measure vortices in superfluid Helium due to some of their intrensic properties. When a vortex is created in superfluid He-4, it has a normal core with a diameter of about $3\mathring{A}$, as described above, and the fluid closest to this core has the highest velocity. Thus, when a thin wire with a diameter much greater than $3\mathring{A}$ is present in the cell, the vortex's lowest energy state is reached when the wire takes the place of the normal core and the highest velocity areas of fluid flow. The vortex then surrounds the wire as shown in Fig. 3 and can alter the wire's motion.

In past experiments one end of a vortex has been observed coming off the wire and extending to the cell wall instead, as shown in Fig. 4. When this occurs, the free piece of the vortex core rotates around the wire. During this rotation, its interaction with the cell wall causes the vortex to lose energy and precess down the cell [2]. This can be observed as oscillations shown in Fig. 6 when the wire itself is off-center. However, when there is a large enough irregularity in the cell wall, the vortex core may pin to that location. When this occurs, new phenomena can be observed, such as vibrations in the vortex core known as Kelvin waves shown in Fig. 4. To encourage pinning, a hemispherical stycast bump was included in the vibrating wire cell used for this experiment. It was placed about halfway down the cell, vertically.

## 2.3 Creating Vortices

Before vortices can be measured, they must first be created. In the past, they were created by disconnecting all the wires from the fridge and physically rotating it in the dewar [5]. Though this method is reliably able to create vortices with a controlled direction, it requires someone to be in the lab to do it. Thus, a newer method was used this summer, in which a vortex was created by exciting the vibrating wire with high amplitude continuous oscillations near its resonant frequency. By varying the current passed through the wire and excitation time, the position of the vortex can be manipulated: if excited longer, the vortex covers the whole wire and remains stable at a quantized state of N=1, but if not, it only extends partway and begins precessing immediately. This method can be used to take overnight measurements of vortices. However, the exact vortex created can not yet be accurately predicted, so overnight measurements can not be adjusted based on where the vortex pinned, if at all. This could be improved in the future by expanding the existing LabView program to recognize if and where a vortex is pinned.

# 3 Measurement Techniques

## 3.1 Pulsed Mode

Once a vortex is caught, its properties are measured by manipulating the wire at its core. This is done using one of two methods: single pulses of current or continuous low amplitude vibrations. In the so-called pulsed mode, the wire is plucked by sending a short current through it while there is a magnetic field present. This pulse moves the wire away from its equilibrium position, and when the current is gone it vibrates as it returns to equilibrium. These vibrations cause an induced voltage which is then measured. The wire's motion is very rapid, but due to its imperfections, it has two resonant frequencies which result in an envelope frequency that can be fit using a LabView program even when there is no helium or circulation present. When there is helium in the cell, the wire's vibrations are damped due to the presence of some normal fluid, and the decay of the envelope oscillations is shown in Figure 5. The beat frequency of these oscillations is also a function of circulation, so it can be used to find the quantum number of a vortex. Vortices caught on the wire are almost always N=1, so the quantum number measured by these decays actually describes what portion of the wire is covered by the vortex.

While the wire motion caused by this mode is well understood, it can only be measured every few seconds. If the wire is pulsed at a higher rate, the pulses can interfere with one another, changing the beat frequency. In addition, the LabView fitting program works best with at least two minima, so the measurements are limited to a minimum of about three second pulses even when the interaction between decays is not significant. Thus, the resolution that can be achieved is severely limited and some high frequency phenomena cannot be observed.

## 3.2 Continuous Mode

The other measurement technique seeks to increase data resolution but is less well understood than the pulsed measurements. In this method, known as sweep measurements or continuous mode, the wire is continuously excited using a sine function generator and its response amplitude is measured. When the wire is excited over a range of frequencies, the response amplitudes take the form of a double-Lorentzian as shown in Figure 7, which has a slight dependence on temperature. The peaks on this graph represent normal resonant modes of the wire, and the distance between these peaks is a function of circulation in the cell. The further apart the peaks are, the more of the wire is covered by a vortex.

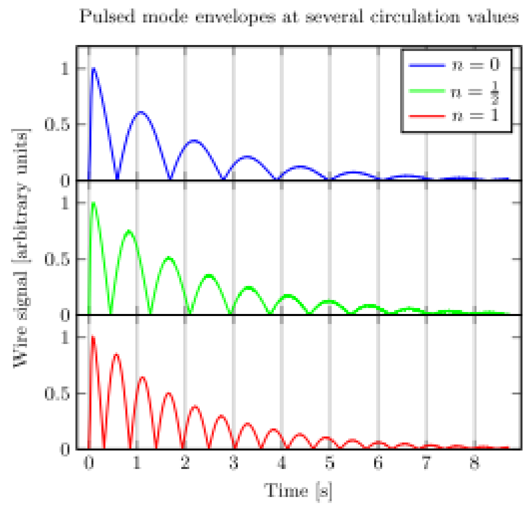


Figure 5: Pulsed mode envelope decays with varying circulation values. In the top data set, there is no circulation while in the bottom two, a vortex covers half and all of the wire respectively. As the circulation increases, the beat frequency also increases. The circulation values can be determined from these decays using a fitting function. [6]

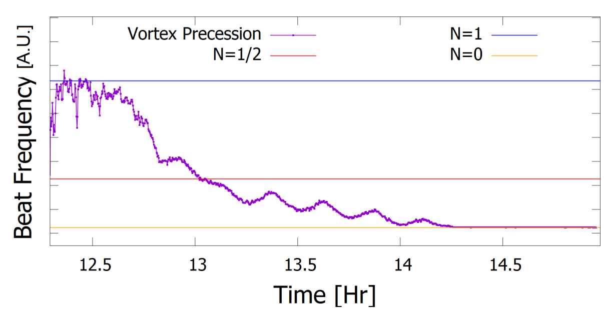


Figure 6: This graph of beat frequency versus time shows the precession of an off-center vortex over time. The maxima of these oscillations correlate to moments when the wire is closest to the cell wall and the most wire is covered by the vortex. These oscillations typically have a period of around 15 minutes.

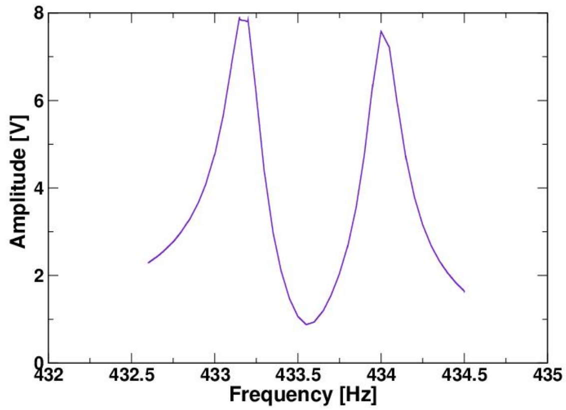


Figure 7: A frequency sweep of the wire when no vortex is present. As the excitation frequency approaches one of the resonant frequencies of the wire, the response amplitude increases forming a peak structure. The separation between these two peaks is a function of circulation.

However, such frequency sweeps take minimally the same amount of time as the pulsed measurements and can often take longer. Thus, it is more useful to examine the change in response amplitude at a certain frequency over time. Measurements are taken on the inside edge of a peak, where the slope is steepest. From there, shifts in the double-Lorentzian peaks can be measured without having to sweep all of the frequencies, and continuous measurements can be made with a higher resolution than the pulsed mode data. In addition, a steady interaction between the wire and the vortex may be easier to account for than the more erratic interaction that occurs in the pulsed mode, where the wire amplitude shifts dramatically over time.

# 4 Results and Discussion

This summer, the main goals were to compare the behavior of the vortex and wire in pulsed and continuous modes and to investigate higher harmonic Kelvin waves than had been observed before. Preliminary trials of a new continuous mode setup with two superimposed oscillation frequencies were also attempted.

## 4.1 Pulsed Mode Findings

In order to investigate higher harmonic Kelvin waves, pulsed measurements were first used to determine whether or not a vortex was pinned. The pulse rate varied, but it was found that the vortex was more likely to pin when a pulse was sent every nine seconds. Higher frequency pulses may cause the end of the vortex to move more, allowing it to get past irregularities in the cell wall, such as the stycast bump.

In pulsed mode, when the vortex pinned at what is thought to be the stycast bump, a characteristic signature was observed repeatedly, though not consistently. This location is believed to be the stycast bump because the vortex pins in that location with some regularity. The shape of the signature is shown in Fig. 8. There is no clear explanation for this pattern yet, though it is possible that it forms as the vortex interacts with the bump to reach the lowest possible energy state. Future work will be done to help explain the vortex's behavior as it pins to the stycast bump on the side of the cell wall.

Once the vortex is pinned, some oscillations in the vortex core can be observed using pulsed mode with nine second pulses. However, when the pulse frequency is increased, the Kelvin waves become clearer and more regular. There were also oscillations with a higher frequency superimposed. These higher frequency oscillations are postulated to be third harmonic Kelvin waves. When the measurement frequency is further increased, an interesting phenomenon begins to occur. Consistently, three second pulses produced a distinct pattern shown in Fig. 9. This pattern is characterized by sharp peaks with smooth dips, and it has a significantly lower frequency than previously observed Kelvin waves on the same vortex. In addition, the Kelvin waves of both frequencies are superimposed over this pattern. A similar pattern was observed with even higher frequency measurements.

Fig. 10 also shows Kelvin waves observed in pulsed mode during precession. The lower frequency Kelvin waves typically have a period of about one minute, while the precession frequency is about 15 minutes. However, higher frequency Kelvin waves were not clearly observed during precession.

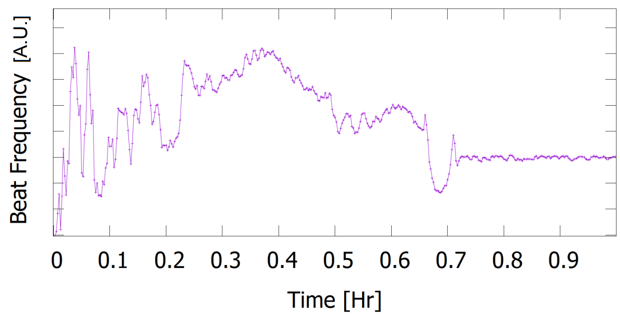


Figure 8: This graph of time versus beat frequency shows a clear signature as the vortex pins on the stycast bump at around 0.65 hours. The beginning part of the graph shows precession followed by a broad dip leading to a nearly horizontal line once the vortex pins. This signature was observed repeatedly when the vortex pinned.

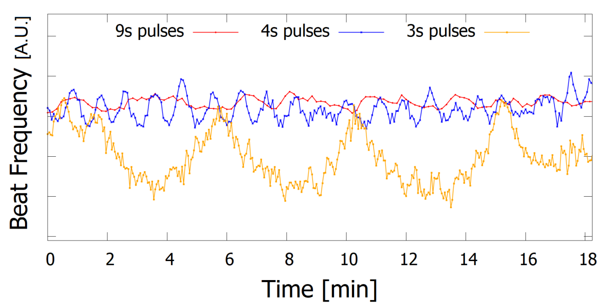


Figure 9: This graph of time versus beat frequency shows the behavior of a pinned vortex in pulsed mode. The red line represents data taken with nine second pulses while the blue line represents four second pulses. The blue line shows clearer Kelvin waves with higher frequency oscillations superimposed. The yellow line shows three second pulses, which result in a clear dip pattern superimposed over the Kelvin waves. The curves are shifted in time to overlap.

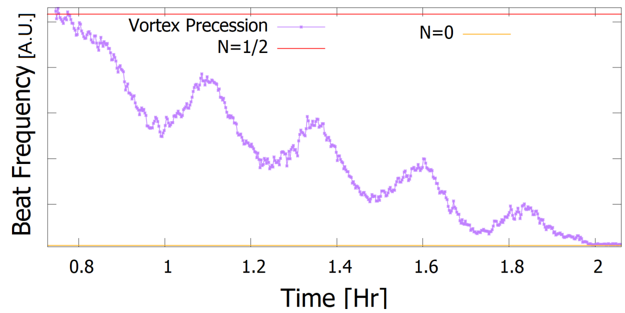


Figure 10: This graph of beat frequency versus time shows vortex precession, with a period of about 0.25 hours, as well as additional higher frequency oscillations that are thought to be Kelvin waves.

## 4.2 Continuous Mode Findings

The vortex's motion was also observed using continuous mode measurements. Precession could be clearly seen using this method. In addition, some precessions had higher frequency oscillations superimposed that may be Kelvin waves as shown in Fig. 11. However, these oscillations had a longer period than expected with Kelvin waves. It was difficult to take more measurements of this phenomenon because the settings required to take meaningful data in continuous mode, such as excitation frequency and amplitude, seemed to change with every vortex and every cooldown. There is much work needed to find the best parameters for consistently seeing Kelvin waves in continuous mode.

In general, the focus this summer was more on pinned vortices than precessing ones. Once a vortex was found to be pinned in pulsed mode, the system was switched to continuous mode. In this mode, it was possible to view Kelvin waves with some higher frequency oscillations as shown in Fig. 11. However, it is unlikely that these faster oscillations were also Kelvin waves because their frequency was higher than would be consistent with such waves. Nonetheless, further data analysis must be done to determine the exact frequencies of these oscillations and whether or not they may be Kelvin waves.

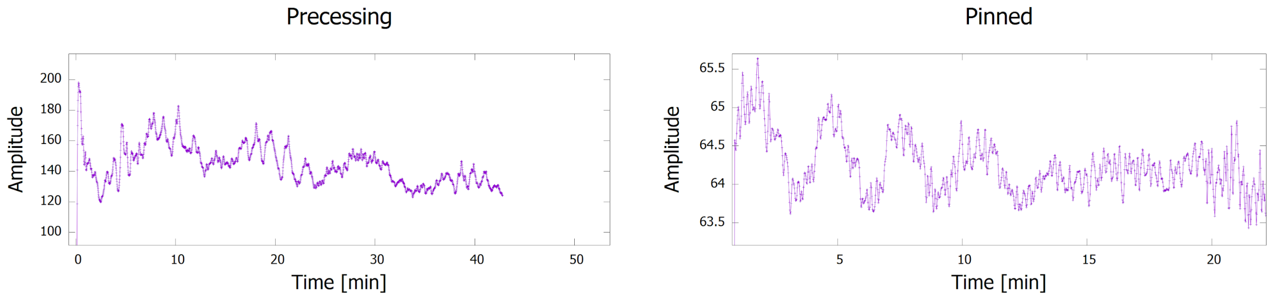


Figure 11: **Left:**The graph of response amplitude versus time shows a vortex precession in continuous mode. The period is consistent with precession in pulsed mode, and precession continues when switching between modes. This precession has higher frequency oscillations superimposed. **Right:**The graph of response amplitude versus time shows a pinned vortex in continuous mode. It features clear oscillations that may be Kelvin waves with higher frequencies superimposed.

In order to more clearly see higher harmonic Kelvin waves in continuous mode, some trials were performed using two different function generators. The goal of these trials was to cause low amplitude oscillations for continuous mode measurements, while simultaneously adding in a lower frequency, higher amplitude oscillation at the resonant frequency of the Kelvin waves. Theoretically, this would make the Kelvin waves more obvious. This technique should be further investigated, as it showed some promise and may in the future be able to isolate higher harmonic Kelvin wave frequencies experimentally.

# 5 Future Work

While many interesting phenomena were observed this summer, they seem to pose more questions than they answer. The first step that should be taken next is analyzing the raw data taken. It will be especially interesting to perform Fourier transforms of the observed oscillations and compare the resulting frequencies. In addition, comparing some of the interesting patterns with three second pulses and during pinning may help to explain these phenomena. Furthermore, it may be possible to view the oscillations in an even higher resolution by fitting portions of the stored decays instead of the entire decay. Reviewing the data may also help determine the best parameters for observing Kelvin waves in the continuous measurement mode. Finally, the patterns caused by the two-oscillator trials should be compared and Fourier transformed to explain the physical reality represented by the data.

In addition to data analysis, the experimental setup can also be improved for future trials. For one, the LabView program used to take data could be automated to detect whether or not a vortex is pinned, and to use this information to decide what kind of data will be taken. This would be especially useful for taking measurements overnight. Without this automation, overnight measurements are often useless. This is because the vortices produced by exciting the wire do not always pin and when they do pin, it is very difficult to predict when it will happen. Thus, when programming the overnight measurements, it is necessary to guess what the vortex will do, leading to a lot of wrong guesses and useless data. Additionally, a three-way switch could be installed in the electronics and programmed into LabView to make it easier to switch between the pulsed and continuous modes. Finally, for future experiments, it may be interesting to develop a three-part cell with a micro-oscillator installed in the middle of the cell instead of a bump. This could then be used to excite Kelvin waves manually and improve measurement techniques.

Eventually, these improvements will hopefully be used to explain how and when Kelvin waves on a vortex core become non-linear and to extrapolate these findings to explain the behavior of a normal fluid vortex.

# 6 Acknowledgements

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