Glauber Model Monte Carlo Simulation of Pb+Pb Collisions

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Abstract

This project focuses on the analysis of a set of data from the Compact Muon Solenoid (CMS) experiment at the Large Hadron Collider located at CERN. A Glauber Model Monte Carlo method is used to model the collision of two Pb nuclei and simulate the production of transverse energy produced by each nucleon-nucleon collision. The measured distribution of transverse energy is compared to the predictions from the model. Events are statistically classified according to their impact parameter, defining the centrality classes and the trigger inefficiencies for peripheral events to be determined. The centrality classification allows comparisons between peripheral and central collisions, where we expect the most central collisions to reach the highest temperatures with the largest QGP volume. This project will allow the study of quarkonia production from small nuclear systems up to the most central heavy-ion collisions in order to disentangle cold nuclear matter effects from those related to the color deconfined Quark-Gluon Plasma.

I. Introduction

Protons and neutrons are not elementary particles, they are made up of even smaller constituents - quarks and gluons. A proton or neutron is each made up of three valence quarks, which are held together by gluons, particles that carry the strong force and hold the quarks together. Quarks are confined within larger particles, thus they cannot be separated and studied in isolation. Furthermore, the strong force between two quarks becomes larger as they move farther apart, whereas the electrical force between a nucleus and an electron, grows weaker as their separation increases. When heavy ions are collided at high energies, a medium of nearly asymptotically-free quarks and gluons forms. This medium is coined Quark Gluon Plasma (QGP) and is considered a new form of matter that interacts differently with particles than normal matter. This is due to its physics being dominated by the strong interaction between color-charged constituents. QGP is a hypothetical, highly energized form of matter that contains unbound quarks and gluons, believed to have been present ten millionths of a second after the Big Bang. Understanding how the transition from the confined state to this Quark-Gluon Plasma (and vice versa) occurs is a fundamental goal of heavy ion physicists, who recreate the plasma by colliding nuclei at ultrarelativistic speeds.

Laboratories such as the European Organization for Nuclear Research (CERN) and the Brooklyn National Laboratory both conduct experiments in order to study this matter. The Large Hadron Collider (LHC), located at CERN in Geneva, Switzerland, accelerates protons and heavy nuclei to nearly the speed of light and the collides them to produce the high energy conditions necessary to form QGP. This project analyzes data from the Compact Muon Solenoid (CMS) detector, one of four detectors at the LHC. The CMS detector is a superconducting solenoid with a magnetic field of 3.8 T and contains a silicon pixel and strip tracker, an electromagnetic calorimeter (ECAL), a hadronic calorimeter (HCAL), and muon chambers.

II. Glauber Model

In a collision where two heavy nuclei, such as lead or gold, are collided at ultrarelativistic speeds, thousands of particles are generated. A lead nucleus is made up of nucleons, i.e. pro-

*Advised by: Professors Manuel Calderón de la Barca Sánchez and Daniel Cebra ton and neutrons, and the particle production is a result of the individual nucleons colliding with each other. Each of these collisions produce energy, which can be directly measured by a detector. Thus, it is natural to ask just how many nucleons are involved in any particular collision, or, more reasonably, in a sample of selected collisions. To answer this question may seem like a difficult feat since the detectors cannot directly observe quantities such as the number of participating nucleons (N_{part}), the number of binary nucleon-nucleon collisions (N_{coll}), nor the impact parameter of the nucleons (b), which is typically on a femtoscopic length scale. However, theoretical techniques have been developed to allow estimation of these quantities from experimental data. These techniques, which consider the multiple scattering of nucleons in nuclear targets, are generally referred to as Glauber models, after Roy Glauber. We implement a Monte Carlo method to create a Glauber Model for Pb+Pb collisions, which is then compared to experimental data from CMS.

III. Method

i. Woods Saxon Density

In the calculations of geometric parameters using a Glauber Model approach, some experimental data must be given as model inputs. One is the nuclear charge density. The nuclear charge density of the nucleon is similar to a hard sphere, but not quite. Therefore, the charge densities are parameterized the Wood's Saxon Density Distribution:

$$\rho(r) = \frac{\rho_0}{1 + e^{\frac{r-R}{a}}} \quad (1)$$

where ρ_0 corresponds to the nucleon density in the center of the nucleus, R corresponds to the nuclear radius, a to the skin depth, and r is distance of a nucleon from the center of a given nucleus. For ^{208}Pb, $R = 1.07$ fm $\cdot A^{\frac{1}{3}}$, $a = 0.54$ fm, and $A = 208$ nucleons. The probability of this distribution is plotted as $r^2\rho(r)$ [1].

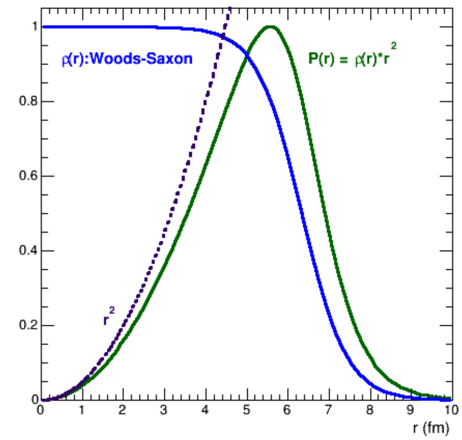

Figure 1: *Figure with the plot of the Woods Saxon distribution in blue. The probability of distribution is plotted in green. r^2 is plotted for reference.*

In the Monte Carlo approach, the colliding nuclei are assembled by randomly distributing the nucleons of nucleus A and the nucleons of nucleus B in a three-dimensional spherical coordinate system with respect to the nuclear charge distribution. Each nucleon is distributed with:

$$P(r,\theta,\phi) = \rho(r)dV = \rho(r)r^2 dr d(cos\theta)d\phi \quad (2)$$

The coordinates of the nucleons are converted from spherical to Cartesian and then plotted on a x-y coordinate system.

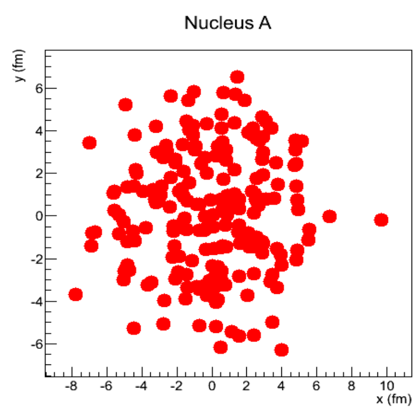

Figure 2: *Model of one Pb nucleus, the nucleons distributed by **Eq. (2)**. The diameter of the nucleons are given by 2R.*

ii. Impact Parameter

The impact parameter (b), is defined as the distance between the centers of the two colliding nuclei in the transverse plane. A random impact parameter is drawn from the distribution $d\sigma/dr = 2\pi r$ from a random number generator, thus the probability of the impact parameter distribution follows a linear trend. The probability of a head on collision ($b = 0$) is much less probable than a peripheral collision where b is very large.

To determine if a binary collision between two nucleons occurs, the following condition must be satisfied:

$$b \leq \sqrt{\frac{\sigma_{inel}^{NN}}{\pi}} \tag{3}$$

where σ_{inel}^{NN} is the total inelastic nucleon-nucleon cross section [2]. For an individual nucleon of a Pb nuclei, σ_{inel}^{NN} is 6.5 fm. An A+B collision is treated as a sequence of independent binary collisions, meaning that the inelastic nucleon-nucleon cross section is independent of the number of collisions that a nucleon underwent previously.

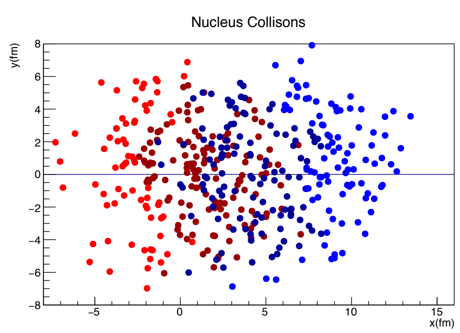

Figure 3: *Model of two Pb nuclei colliding; nucleus A and nucleus B moving along z-axis, perpendicular to the plane. Each colliding (darker-colored) nucleon is considered a participant.*

For each pair of nucleons (one from nucleus A and one from nucleus B), there is a check to see if the distance satisfies the condition in **Eq. (3)**. Any nucleon that collides is considered a participant and is counted for a overall value designated as N_{part}. Each participant nucleon is colored a darker color for ease of visualization that the spectator nucleons, or the nucleons that do not participant in a collision, shown in **Fig. 3**. The collisions are counted up and are designated as the value N_{coll}.

Using the model, 10^6 nucleus+nucleus collisions are simulated. The random impact parameter is drawn from the distribution P(r) proportional to r. For each collision, N_{part} and N_{coll} are calculated. For each event where $N_{coll} > 1$, histograms are filled with the values of N_{part}, N_{coll}, and b, respectively.

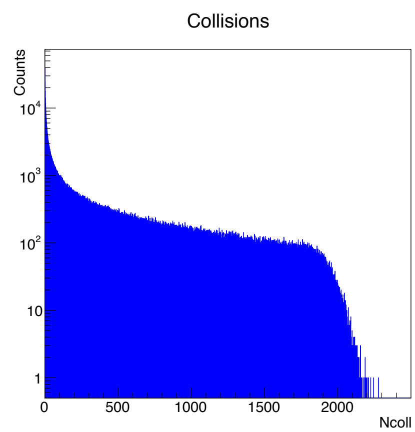

Figure 4: *Histogram representing N_{coll}.*

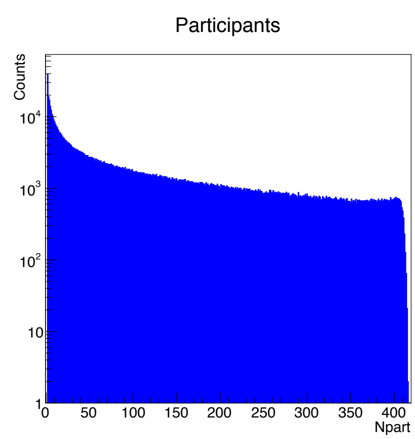

Figure 5: *Histogram representing N_{part}.*

Neither N_{part} nor N_{coll} can be measured directly in the CMS experiment, therefore a model of an experimental observable, the transverse energy, needs to be created in order to relate the data to the model. Each nucleon-nucleon collision produces particles and this particle production produces energy measured by the HCALs of the CMS detector. This particle production can be modeled using a negative binomial distribution. In previous p+p experiments, such as UA5, the number of hadrons produced in a given collision followed approximately a negative binomial distribution. Therefore, for parameterizing experimental data, it is a useful approximation [3].

iii. Negative Binomial Distribution

In a sequence of independent Bernoulli trials where the probability of success in each trial is p, the negative binomial distribution (NBD) describes the probability of k failures occurring before n successes occur. The probability mass function of the NBD is defined as:

$$f(k) = \begin{pmatrix} n+k-1 \\ n \end{pmatrix} (1-p)^k p^n \tag{4}$$

where the mean of the NBD is $\mu = \frac{pk}{(1-p)}$, where p is the probability of failure in each trial.

When writing the NBD in terms of its mean instead of p and extending to real values of n, the NBD probability has the algebraic equivalency of:

$$f(k) = \frac{\Gamma(k+n)}{k!\Gamma(n)} \cdot \frac{(\mu/k)^n}{(1+\mu/k)^{n+k}} \tag{5}$$

The parameters k and n will be used as the parameters of the negative binomial. The transverse energy distribution is obtained by convolving the NBD with the N_{coll} histogram. This mathematical convolution physically represents that a given nucleon-nucleon collision is the source of particles, which produce the transverse energy. The energy produced in each nucleus+nucleus collision is modeled by sampling from the NBD N_{coll} times. In this convolution, N_{coll} is obtained from the Glauber Model calculation for that particular nucleon-nucleon collision, which gives one realization (pseudo-event) for that value of N_{coll}. A distribution for that particular value of N_{coll} is obtained by repeating the same step. The histogram for the sum of the transverse energy (SumEt) is built by summing the results for all values of N_{coll}. The resulting histogram is given by **Fig. 6**.

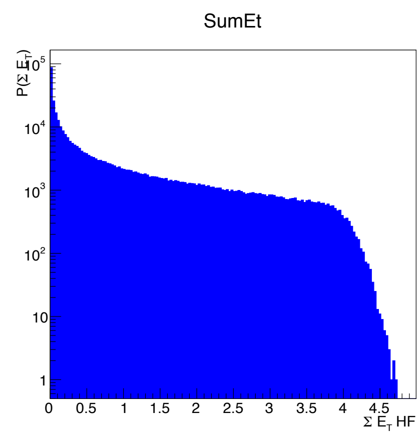

Figure 6: *Histogram representing SumEt obtained from N_{coll} histogram and NBD.*

The SumEt histogram with arbitrary NBD parameters $k = 0.96$ and $n = 1.6$ is overlaid with a histogram containing the experimental data and the fit of the two is assessed. The NBD parameters are manipulated via a χ^2 test in order to obtain a better fit.

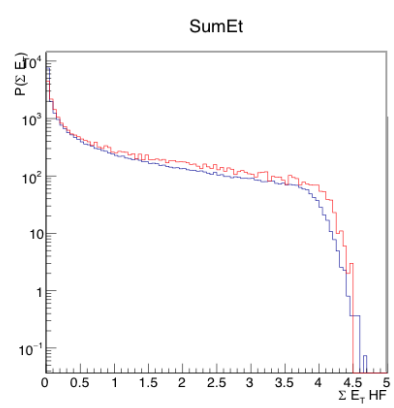

Figure 7: *Obtained the red curve using the N_{coll} distribution and convolving it with the NBD, fit with primary NBD parameters. Blue line represents CMS data [4].*

iv. Fitting Parameters

The parameters of the NBD function must be adjusted so that the SumEt histogram fits to the experimental data. The NBD parameters that produce the lowest χ^2 value will produce the best fit. For the Glauber Monte Carlo, the 2-D parameter space is scanned and the resulting χ^2 value for a given parameter is represented by a colored 2-D plot. The parameters are in 2-D axes in which the height (color) represents the χ^2 values for that choice of parameters. The parameter space is continuously scanned until the minimum value is at the center. The final pass produced a χ^2 value of approximately 100.

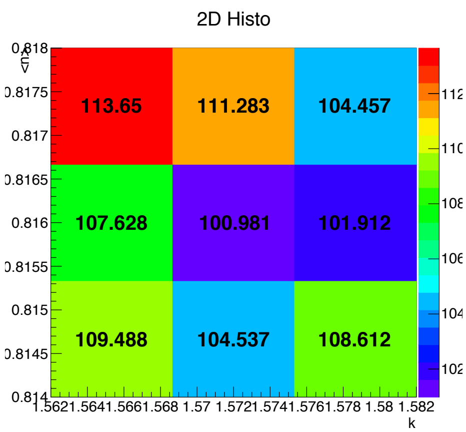

Figure 8: *χ^2 scan of the 2-D parameter space. Parameters were zeroed in on, and obtained a reasonable χ^2 that had about 100 degrees of freedom.*

Table 1: *Final NBD Parameters*

k	n
1.572	0.816

IV. Centrality Classes

Centrality classes assume the that impact parameter is related to particle multiplicity. For large b (peripheral collisions), low multiplicity is expected and at small b (central collisions), high multiplicity is expected.

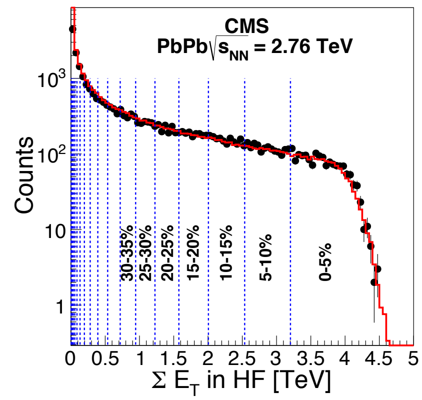

Figure 9: *SumEt histogram (red) fit to CMS data (black) using NBD parameters from **Tab. 1**. Centrality classes are cut from the fit.*

Table 2: *Centrality Cuts*

Percent Centrality	N_{part}-CMS	N_{part}
$0-5\%$	383	381
$5-10\%$	331	329
$10-15\%$	284	283
$15-20\%$	242	240
$20-25\%$	205	203
$25-30\%$	173	171
$30-35\%$	144	142
$35-40\%$	120	117
$40-45\%$	98.4	95.8
$45-50\%$	79.7	76.8
$50-55\%$	63.2	60.4
$55-60\%$	49.1	46.7
$60-65\%$	37.5	35.3
$65-70\%$	27.7	25.8
$70-75\%$	19.6	18.5
$75-80\%$	13.4	12.8
$80-85\%$	8.76	8.64
$85-90\%$	5.45	5.71

This is shown in **Fig. 9**, in which the SumEt histogram is fit to the experimental data histogram using the NBD parameters (Tab. 1) that gives the lowest χ^2 value. The data are only fit from 0.5 TeV to 5 TeV instead of 0-5 TeV to avoid the inefficiencies in peripheral collisions. Events that are most central are determined by making centrality cuts. For each histogram, centrality classes are defined in terms of a fraction of the total integral. The centrality classes are recorded based on the bin values of the histogram for when the fraction of the integral equals 5%, 10%, 15%, etc. Once a centrality class is defined in the model, the mean value of Npart can be calculated for all events that fall in that centrality bin. The centrality classes are cut on the histogram containing the experimental data and SumEt histogram.

Tab. 2 lists the mean values of N_{part} calculated for each centrality class. The CMS experiment made a similar model and calculated the mean values of N_{part} as well [4]. For comparison, their data set is shown, in addition the values calculated from this model.

V. Discussion

For Pb+Pb collisions, QGP is expected to be created. For p+Pb collisions, there are not enough nucleon-nucleon collisions to produce QGP. Therefore, in p+Pb collisions, cold nuclear matter effects are studied. The comparison between p+Pb and Pb+Pb collisions is used to determine how much of the effects in Pb+Pb are due to the Quark-Gluon Plasma. The Glauber Model determines centrality classes, which are used to study physics observables from the lowest N_{coll}/N_{part} to the largest N_{coll}/N_{part}. The lowest N_{coll}/N_{part} is studied in p+Pb collisions, where QGP is not expected to form, and the largest N_{coll}/N_{part} is studied in Pb+Pb collisions, where QGP is expected to form. This project gives insight into one part of this final goal. The next step of this project is to create a Glauber Model for p+Pb at the same energy, which contributes to the long term study that will compare hot vs. cold nuclear matter effects in order to disentangle Quark-Gluon Plasma physics.

References

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[4] CMS: HIN-10-001, HEP 08 (2011) 141.