

Effects of Temperature Changes on a Vibrating Wire in Superfluid Helium

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This work describes the experimental set up to study vortices in superfluid Helium 4. Analysis of data from the summer of 2016 has revealed responses in the behavior of a vibrating wire in superfluid due to changes in temperature. Explanation of these observations, as well plans for future experimentation are included in this work.

I. INTRODUCTION

A. Vortices

Research on vortices, defined as regions of fluid rotating around an axis, has many practical uses.[4] This research can aid in the study of natural phenomenon such as hurricanes, tornadoes, and whirlpools. Vortices can also be observed in smoke rings and the air around ones mouth when whistling. The turbulence off of airplane wings is yet another example. Vortices created by large planes can cause problems for a smaller aircraft nearby. An understanding the properties of these vortices can lead to the invention of safer airplanes. Another application is the understanding of the rotation of neutron stars.[4] Once vortices are understood, not only will these examples be better understood, but there will be a potential to create and improve vortex applications.

B. Superfluids

When studying vortices, we observe them in superfluids, or a fluids with zero viscosity. In a normal fluid, circulation can occur throughout the fluid. Therefore, not all circulation present in the fluid is necessarily a vortex. This allows for the creation of complex systems of vortices. In addition, vortices in a normal fluid have the potential to change in core size and rotation speed due external forces. The viscosity of a normal fluid allows for energy dissipation which can slow the circulation of the vortex. With a vortex changing in size and speed, it can be extremely difficult to understand its properties and what factors may affect it.

While superfluid itself has zero circulation, our use of a vibrating wire allows for quantized circulation confined to the wire. Due to this quantization, the circulation in our experiment stays relatively constant. One property of a vortex is that its core must be able to rotate, requiring normal fluid. Therefore, a vortex cannot occur in pure superfluid. In our experiments, we use a thin superconducting wire that acts as the vortex core. Another property of superfluid vortices is that the vortex lines must extend to the boundaries of the container. This means that our vortices will always extend to the walls of our experimental cell. Once superfluid vortices are understood, a better understanding of vortices in normal fluid can be achieved. In our experiment, we study vortices in superfluid Helium. In order for liquid Helium to become a superfluid, it must reach low temperatures. The transition temperature for

Helium 4 is approximately 2.17 Kelvin. This is much higher than that of Helium 3, appx. 1mK, making it a better choice.

C. Cooling the Fridge

In order to create superfluid, we use a three step cooling process. While Helium 4 transitions at a temperature of around 2K, we usually take measurements closer to 300mK. The lower temperature increases the ratio of superfluid to normal fluid. Initially, the fridge is submerged in liquid Helium. This is done by sliding the entire fridge through the neck of a Helium 4 dewar using a crane. The contact between the vacuum can enclosing our experiment and the liquid Helium, as well as the addition of exchange gas in the can, create a thermal link between our experiment and the Helium. The liquid Helium 4 will cool the fridge to approximately 4.2 Kelvin, the boiling point of Helium 4. The next two steps are processes of evaporative cooling.

The second step in the cooling process is Helium 4 evaporative cooling. A long siphon line extends the length of the fridge and collects Helium 4 from the dewar. It siphons the Helium up into what is referred to as the 1K plate. A mechanical pump connected at the top of the fridge pumps on the 1K plate to remove the gas above the liquid helium. This pumping process forces evaporation of Helium, and thus cooling of the 1K plate. The process brings the 1K plate to a temperature of approximately 1.5 Kelvin. The third cooling step involves evaporative cooling of Helium 3. Below the 1K plate on the fridge is the Helium 3 pot. Above the 1K plate is a charcoal container that aids in the evaporative cooling process. When the charcoal approaches the boiling point of Helium 3, it begins to absorb Helium 3 gas. In this way, it acts as a pump. Similar to the last step, the process forces evaporation and cooling of the Helium 3 in the pot. This evaporation gets the Helium 3 pot to a temperature of around 300mK. Helium 3 is rarer than Helium 4 and is thus much more expensive. For this reason, we have a finite amount of Helium 3 in the fridge that is not altered. When the charcoal has absorbed all of the Helium 3, the cooling stops, and a restart occurs. During a restart, the charcoal must be heated to release the Helium before the fridge is cooled again. This can greatly limit the time available for taking measurements.

II. EXPERIMENTAL SETUP

A. The Fridge

A diagram of the fridge with labels can be seen in Figure 1. The siphon line can be seen on the left running up the length of the fridge and into the 1K pot. Below the 1K plate is the Helium 3 pot where the final cool down step occurs. A fill line starts at the top of the fridge, goes through the 1K plate, and spirals down the fridge. After passing through a hole in the Helium 3 pot, helium sent through the fill line will enter our experimental cell. As the helium travels through the hole in the Helium 3 pot, it is important to note that the helium 4 filling the cell does not mix with the helium 3 in the fridge. The cell, as described in the following section, is where our experiment occurs. Two magnets in a cylinder around the cell create a magnetic field. This field is used when we take measurements. In addition, there are wires running the length of the fridge. These wires, allow us to interact with the vibrating wire, the magnets' persistent current switches, and allow us to measure the temperature at different points. These wires are attached by various solder joints. To thermally isolate the fridge from the liquid Helium 4 it is submerged in, we have a vacuum can that encloses the fridge. This is put on with a mechanical pump and vacuum grease to make a leak tight seal.

When running the fridge, there are many problems that can occur. One problem that we encountered was the presence of air in the siphon line. This air froze when we cooled down, creating an ice plug in the line, and preventing us from opening the needle valve at the top of the fridge. The opening of the needle valve allows us to put helium through the siphon and cool the 1K plate. Therefore, the presence of the plug made us unable to condense helium into the cell. We also encountered a leak in the vacuum can. This was due to a dent in the side that needed to be fixed in the machine shop. In addition to problems with the physical components, there is a high risk of electrical problems. Frequently one of the wires connecting the cell to the fridge, or the wires running up the fridge, would break or short. This resulted in troubleshooting and soldering of wires. These electrical difficulties also extended to the equipment used to interact with the fridge. We had cables and control boxes that would have breaks or shorts in need of fixing. In one such instance, we remade a cable that connects two 12 wire cables to a 24-pin connector.

B. The Cell

The cell, as mentioned earlier, is where the experiment occurs. The cell can be seen in Figure 2. A brass cylinder 2in in length is where superfluid is created. Often, a small stycast glue bump is inserted along the inside wall of the brass. This provides a location for vortex pinning, a concept described later in this paper. A thin superconducting wire is threaded through the cylinder. To create the wire, we start with a thick copper coated wire. After scraping 2in of formvar insulation in the middle of a long strand, we soak it in nitric acid to

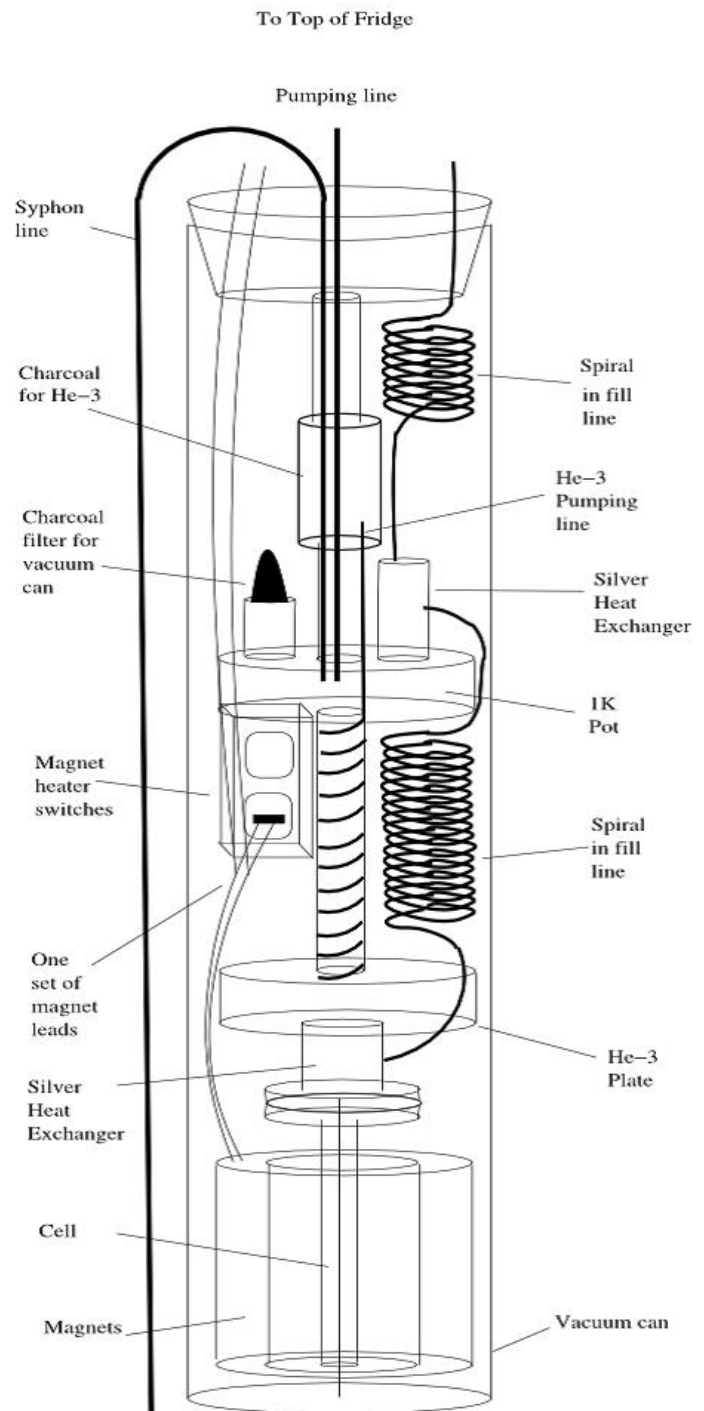


FIG. 1: A schematic diagram of the fridge and its parts. The siphon line along the left runs up the fridge and then down into the 1K pot. A fill line for the cell starts at the top and winds down the fridge. The cell itself is the small cylinder at the bottom of the fridge. Two magnets make up the cylinder that slides up over the cell. A vacuum can encloses the entire set up.

remove the copper. The result is a thick bundle of 187 superconducting wires. These wires are twisted together and must be carefully cut so that only one of the 17 micron diameter wires remains. The cut wire is very fragile and is at risk of breaking throughout the process of cell assembly. Once the wire is inserted into the cylinder, it is sealed in with two stycast caps. The wire is then stycasted into place while under tension of a hanging weight. If the stycast is not properly set, or the pieces are not secure, leaks can occur. A cell that leaks cannot be used and must be reassembled. Another common issue we encountered was having a crooked cell. This could be due to any number of pieces being bent or crooked. Often this occurred when one of the caps was stycasted at an angle. If the cell was not straight enough, we would be unable to place the magnets over the cell and fit the vacuum can without risking a thermal connection. In one instance, we broke a stycast cap in half while trying to straighten out a cell on the fridge.

A brass piece called the base connects the cell to the fridge with an indium O-ring and screws. This piece has a small copper-nickel tube that connects to a stycast block. The wires fed through this tube are soldered to the superconducting wire and used to connect the wire to the fridge electronics. The other end of the wire can be seen coming up from the bottom of the cell in Figure 2. This piece can cause many problems during cell assembly. For example, the base could leak. A leak is often the result of a problem with the solder connection between the brass piece and the copper-nickel tube. Another problem that frequently occurred this summer was the shorting of the wires that connect the cell to the fridge. This results in a shorting of the vibrating wire to ground, making the cell unusable.



FIG. 2: A picture of the cell. The length of the brass cell is approximately 2-3in. A thin superconducting wire goes through the cell and comes out the stycast caps as a thick stranded wire. The top piece, known as the base, consists of a brass piece, copper-nickel tube, and stycast block. The tube and block allow the connection of the wire to the fridge. The base uses an indium O-ring and screws to secure the cell to the fridge.

C. Creating Vortices

Once all the pieces of the cell and fridge are assembled, we run leak checks and cool the fridge down. After cool down, when the cell is filled with superfluid, vortices are created. This is done by manually rotating the entire fridge in the dewar. Wires are able to detach and then be reattached after the rotation. Before a vortex is present, the circulation of the cell is what is called $N=0$. This can be seen in Figure 3. When a vortex is created, the wire will act as its core. This is called $N=1$, where the vortex spans the length of the wire. Due to the properties of a vortex in a superfluid, the vortex line will span the entire length of the cell. Sometimes, one end of the vortex will detach from the wire. We call this the free end. The free end will move with the circulation around the cell, a process referred to as precession. Often, the wire is off center in the cell. As a result, the free end will have to change its length in order to extend to the wall of the cell, causing the end on the wire to change in length. When taking measurements, as is described in the next section, the extent of wire covered by a vortex can be measured. During the cell assembly process, a bump is inserted on the wall of the cell. The bump creates a site for vortex pinning. When pinning occurs, the free end attaches to the bump, ceasing its precession around the cell. Previous research has seen the occurrence of oscillations in a pinned vortex called Kelvin Waves. [5]

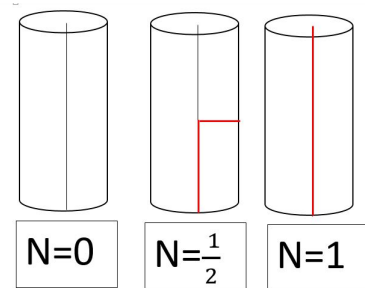


FIG. 3: A schematic of the circulation in the cell. The red line indicates a vortex. In the instance of $N=0$, there is no vortex on the wire. $N=1$ happens when there is a vortex spanning the entire wire. The third image, $N=1/2$ illustrates the case in which one end of the vortex has detached from the wire and is precessing around the cell.

III. MEASUREMENT TECHNIQUES

A. Pulsed Measurements

There are two different methods we can use to take measurements, the first of which is pulsed measurements. The measurement process utilizes the two magnets that surround the cell. There are two magnets, an inner magnet and an outer magnet. The fridge electronics and the vibrating wire create a closed wire loop. Due to presence of the magnets,

there is a magnetic field going through the loop. A function generator is used to send a pulse of current through the superconducting wire. As a result, the wire experiences a Lorentz force, and moves away from its equilibrium position. When placing the wire in the cell, it is secured under tension. Due to the tension, the wire will seek to return to equilibrium and will vibrate when the pulse ceases. The frequency of the vibration is typically around 540Hz. When this movement occurs, due to the magnetic field, an induced voltage (emf) is produced. This emf is what we measure. The measured voltage changes depending of how much of the wire is covered in a vortex, allowing us to see if we successfully trapped a vortex on the wire. Figure 4 shows an example of data acquired from a pulsed measurement. These data form an envelope of the wire's motion over time due to a pulsed current. The actual motion of the wire is small oscillations that occur within the envelope. As seen in the envelope, there is a decrease in the response amplitude over time. The decrease results from the normal fluid in the cell that is damping the wire.

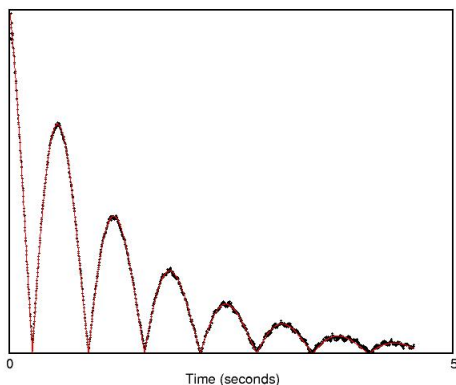


FIG. 4: An envelope generated from a pulsed measurement.

The x-axis is time in seconds and the y-axis is the wire's response. A maximum in the response correlates to wire vibration perpendicular to the magnetic field. This response is proportional to the wire's velocity perpendicular to the magnetic field. The curve seen in this graph is not the direct motion of the wire. Instead, it is an envelope that shows the vibration amplitude perpendicular to the field.

B. Sweep Measurements

We can also take sweep measurements. With a sweep measurement, a continuous signal is sent at a given frequency or a range of frequencies. The response of the wire is similar to that of a driven oscillator. [2] If the wire's response amplitude is measured over a range of frequencies, the results show a double peak structure as seen in Figure 5. The two peaks correlate to the normal modes, or resonant frequencies, of the

wire. Each mode correlates to a direction in which the wire will vibrate. The two modes are perpendicular to one another. They can be thought of as the forward to backward movement and left to right movement when observing the wire as a vertical line. The relative peak heights depend on the position of the magnets. For example, if the magnets are aligned so that the magnetic field is perpendicular to one mode, that mode will be excited more and have a higher peak. This resulting curve has a double-Lorentzian shape given by,

$$\frac{2[a^2 + (\omega - \omega_0)^2 + (d\omega)^2]}{[a^2 + (\omega - \omega_0 + d\omega)^2][a^2 + (\omega - \omega_0 - d\omega)^2]} \quad (1)$$

In the above equation, ω_0 is where the double-Lorentzian is centered. [3] The value of $2d\omega$ in the equation correlates to the separation of the peaks. There are a few problems with using the above equation as a fit. For example, the minimum point between the peaks is much lower than would be expected by a typical double-Lorentzian. Typically, the double-Lorentzian would be able to be modeled by combining two Lorentzian curves. However, when applying this method to the data collected in 2016, we noticed that the curves do not fit the peaks very accurately. In order to improve this, further work must be done. The peak separation, $2d\omega$ depends on whether or not a vortex is present and provides another way to identify the circulation. A larger separation indicates a larger area of wire covered by a vortex. Using data from the summer of 2016, we found that the separation for $N=0$ is approximately 1 Hz, the separation for $N=1/2$ is approximately 1.2 Hz, and the separation for $N=1$ is approximately 1.8 Hz. Analysis of graphs for these different circulations shows that the peaks do not separate evenly. That is, the higher frequency peak seems to move out more than the lower frequency peak as the amount of wire covered increases.

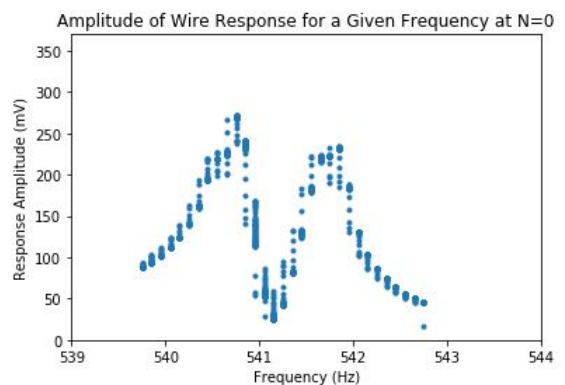


FIG. 5: The result of a sweep measurement at $N=0$. The graph shows the frequency of the excitation versus the amplitude of the wire's response. The two peaks correspond to the two modes of the wire, where their relative heights depend on the alignment of the magnets.

IV. RESULTS AND DISCUSSION

Our original goal this summer was to get a working cell in order to investigate properties of the wire and vortices. One particular area of interest was Kelvin Waves. We also wanted to look at questions such as the strength of vortex pinning and how a wire can excite circulation. We also discussed mapping the change of modes as a function of temperature with and without superfluid in the cell. However, due to the fact that we were unable to get the experimental set-up to work, we analyzed data from last summer. Our set up troubles included the breaking of wires, the leaking of cells, and the breaking of components on the fridge. Some of these difficulties were described in earlier sections of this paper.

A. Trends in Frequency Peaks

One thing we looked at this summer was the change in the frequency peaks' locations over time. When they were taking measurements, last summer's students thought the frequency at which the peaks occurred was changing over time. They would sweep to see the peak pattern, and then sit at a frequency just inside one of the peaks to observe the amplitude as they changed other parameters. Over time, this amplitude changed significantly, possibly indicating a peak shift. In order to look into this, we graphed multiple sets of data from June and July 2016 that had otherwise identical conditions. Using graphs generated in Python, we recorded the frequencies at which the two peaks occurred. Upon initial glance, the graphs appeared as if the peaks did not change. However, when we recorded the frequencies, we noticed changes in location of about 0.1-0.2 Hz. Given the fact that the peaks themselves are approximately 1Hz in width, a shift of that size would have a large impact on the response amplitude. For example, we have the graph in Figure 5. In this graph, a shift of 0.2 Hz to the right would change the response amplitude at 540.75 Hz from around 228 mV to around 218 mV.

We have yet to determine the reason for this shifting. The change in peak location could indicate a change in the wire's resonant frequencies. That would imply that properties of the wire such as the effective mass or tension were changing. Due to our experimental set-up, this is impossible. The wire itself is sealed in a leak tight cell within a vacuum can that is inside the dewar. We could fix the problem in our data by making changes to the LabView program so it will adjust its position. However, further work must be done to understand the reason behind these shifts.

During this analysis of the data, we also wanted to verify that other variables were not affecting the frequency at which the peaks occurred. We predicted that changes in variables such as the excitation amplitude, the amplitude of the signal used to excite the wire, would only affect the response amplitude of the peaks. This hypothesis turned out to be true. In our graphs, the scale of the y-axis differed depending on the voltage used to excite the wire and the voltage divider used. A data set with the y-axis in the range of around 100 mV had a larger excitation voltage than a data set with the y-axis in the

range of 1 mV. An example of this is shown in Figure 6. This confirmation allows us to reliably compare the peak locations over time for graphs of differing excitation voltages.

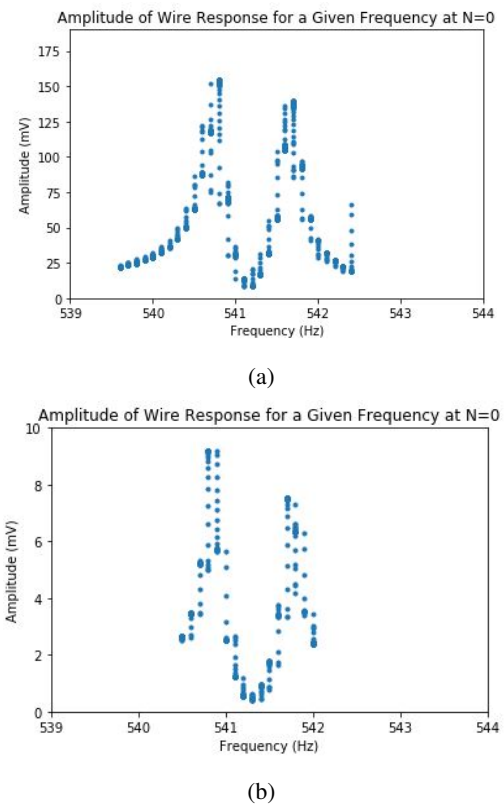


FIG. 6: Frequency sweeps at $N=0$ taken during June 2016. The wire in Graph a was excited with an excitation voltage larger than that of Graph b. The result is the higher response amplitudes seen in a upon comparison with b.

B. Effects of Temperature Changes

Another thing observed by last year's students was the effect of temperature changes on the response amplitude. When they changed the fridge temperature steadily, they saw very little change in the response. However, when they oscillated or changed the temperature rapidly, they observed puzzling results. After observing these effects, they decided to try other things with the measurements instead of investigating further. We looked back upon this data and graphed the sweep measurements.

We began by graphing all files that took sweep measurements at a steady frequency of 541.9Hz. All the files graphed had a circulation of $N=1/2$ and used the same excitation voltage and voltage divider. The data files used had changing temperatures, either oscillations or abrupt changes. The resulting graphs of response amplitude as a function of temperature did not match what would have been expected. It was expected that the temperature change would have a steady effect. However, the results showed unusual patterns as seen the Figure 7.

In the Figure, there appears to be a general downward trend. The peak before the downward sweep is unusual. Similar patterns appeared in the other data graphed and in data of other circulations, signifying that is indeed a trend. The appearance of similar strange patterns for $N=0$ indicates that the result does not depend on the termination of the free vortex on the wire, but rather the wire itself or the surrounding fluid.

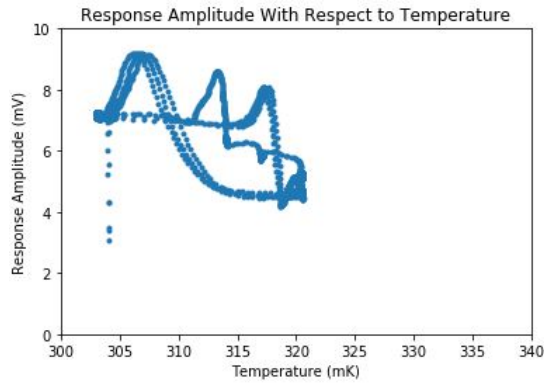


FIG. 7: A graph of the wire's response amplitude with respect to temperature for $N=1/2$. The temperature in this data set oscillated between 305 and 320mK, resulting in an unusual wire response. There appears to be a general decrease in the response amplitude as the temperature increases. However, there also appears to a peak each time the fridge begins to warm.

The graphs similar to those in Figure 7 were instances of abrupt temperature changes. This can be observed in graphs of the temperature with respect to time. To get a better understanding of the data, graphs were made to compare the change in temperature over time with the change in response amplitude over time. An example can be seen in Figure 8. The data set here is the same data from Figure 7. In this Figure, the peaks seen earlier appear as peaks when the temperature changes. It also shows that the response amplitude is generally larger at lower temperatures.

In an attempt to further understand these strange patterns, sets of data with the same temperature oscillations were analyzed. For these data sets, if the data included sweeps between different frequencies, only the data at a specific frequency was graphed. These graphs were very different than the graph shown above. We believe this difference was due to the steady oscillations as opposed to abrupt changes in temperature. Records showed the data as having had very similar parameters. The resulting graphs appear to be banded loops that vary in size. Within the loops themselves, there appear to be small oscillations. These features can be seen in Figure 9.

After graphing the different data sets for the changes in temperature over time, we were able to make conclusions about the effects of different oscillations. The temperature oscillations ranged in speed and magnitude despite having had similar temperature ranges. The different oscillations yielded different sized loops in the graphs of response amplitude as a function of temperature. In addition, some loops had overlap-

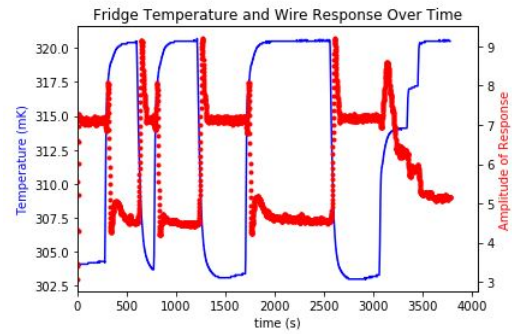
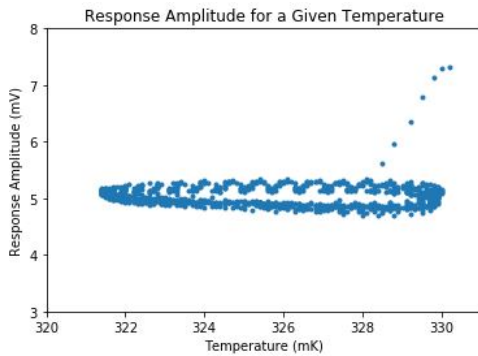


FIG. 8: A graph of both the temperature changes over time and the response amplitude over time for temperature changes at $N=1/2$. In order to have the amplitude and temperature on the same scale, the response amplitude was multiplied by 3 and then added to 300. From this graph, we can see that a higher temperature has a lower temperature response. We also see unusual peaks and dips occurring at the times where the temperature begins to change.

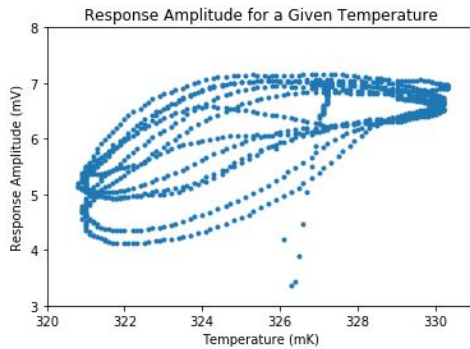
ping bands where others had more spread out bands. From the data, we were able to conclude that slower oscillations yield greater variations in the amplitude of the wire's response. In other words, the loop generated from graphing response versus time is larger for slow oscillations. This is unusual when considering what was previously seen. After the results from the abrupt temperature changes, and the 2016 summer's students observations that steady changes in temperature had little effect, we predicted that the faster oscillations would have had a greater effect. The graphs in Figure 9 disprove this hypothesis. Graph b had oscillations over a larger temperature range than Graph a resulting in the larger range in response amplitudes.

As was done with the abrupt changes, the change in temperature over time was compared with the change in amplitude over time. This comparison allowed us to make multiple observations about the effect of temperature oscillations on the wire. The temperature oscillations were sinusoidal, leading to an expectation of a sinusoidal response amplitude over time. While the pattern appeared to be a sinusoidal inverse of the temperature upon graphing, closer inspection revealed that the wire's response in non-sinusoidal. In addition, there appears to be a phase lag in the response when compared to the temperature. Comparison of data sets in which the temperature oscillations changed in speed revealed noticeable differences in the phase shift. The fast oscillations had a greater phase shift than the slow oscillations. These results can be observed in Figure 10. The data used for Graphs a and b in Figure 10 is not the same data as seen in Figure 9. Both graphs are of oscillations at 541.9Hz for $N=1/2$. The temperature oscillations in the data for Graph b are faster than those for Graph a. The greater phase shift observed in b led to the conclusion that faster oscillations have a greater shift.

The graphs of temperature and response amplitude over time also revealed observations about the overall response am-



(a)



(b)

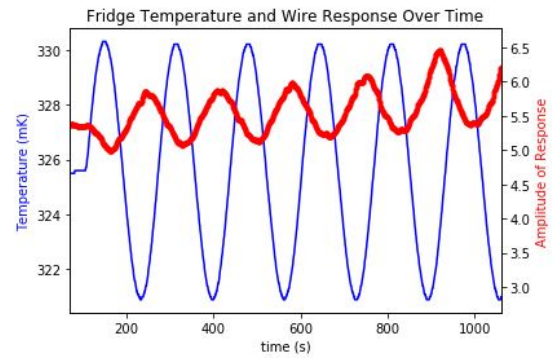
FIG. 9: Graphs of the wire's response amplitude with respect to temperature oscillations at $N=1/2$. The data sets had the same wire excitations. The temperature was approximately 325 ± 5 mK. In b, the oscillations were around three times faster than those in a. Comparison of these graphs led to the conclusion that slower oscillations have a greater effect on the response amplitude.

plitude. As seen earlier, a smaller response was observed for faster oscillations. In addition, the response amplitude rose over time. For sweeps in which the data ran for a longer period of time and oscillated steadily throughout, the amplitude noticeably increased. This can be clearly observed in Figure 10 Graph a.

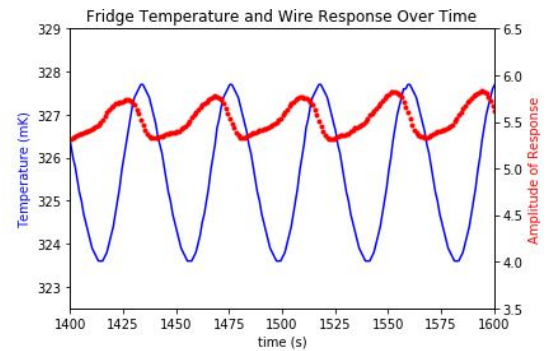
Further graphing of data also revealed that slight temperature changes appear to have little effect on the response. The data used had a temperature of 420mK with 1mK oscillations. The lack of effect could also be altered by the taking of data at a higher temperature than other sets analyzed. In addition, the data files for these temperatures were significantly smaller than others used. Therefore, before we can make definitive conclusions from this data, we need to take and analyze more measurements.

V. CONCLUSION AND FUTURE WORK

In conclusion, we ended the summer with many questions. Our work has led to the discovery many interesting things in



(a)



(b)

FIG. 10: A graph of both the temperature changes over time and the response amplitude over time for temperature oscillations at $N=1/2$. In order to have the amplitude and temperature on the same scale, the response amplitude was multiplied by 3 and then added to 310. The oscillations in Graph a were slower than those in Graph b. These graphs illustrate the inverse, non-sinusoidal response. Graph b has a greater phase shift than graph a, illustrating that a faster oscillation yields a greater phase shift.

the data. However, at this time, we do not understand why these unusual findings are occurring. Until it is understood why the wire responds to temperature as observed, we can not determine how a vortex responds to temperature changes. The understanding of the wire can ultimately get the experiment to a point where Kelvin Waves, and other vortex properties, can be understood. We have made a connection between the speed of temperature oscillations and the amplitude and phase of the wire's response amplitude. As mentioned above, a slower oscillation has a smaller phase shift and a larger amplitude. We have also noticed an increase in the wire's response amplitude over time in the presence of temperature oscillations. These findings, as well as the patterns occurring in abrupt temperature changes, must be investigated further. The first step is the taking of measurements that can increase the amount of useable data. For example, taking multiple measurements of temperature oscillation with similar conditions would be ideal. By changing the speed and range of oscillation, more can be learned from the data. This data should be taken at different circulations as well. Most of the data we looked at

this summer had a circulation of $N=1/2$. It would be helpful to look at $N=0$, as well as $N=1$, to see if the presence of a vortex has an impact. Another thing we were able to confirm was the shifting of peaks over time. In order to understand these shifts, it would be helpful to know exactly how much the peaks shift over time. To do this, we need a more accurate graph. This would be done by fitting the double-Lorentzian and calculating the peak. Once the data is fit, the scale of shifting can be compared to other properties such as shifts in temperature and changes in circulation. However, due to the problem with the Lorentzian fit, as mentioned earlier in this paper, we are unable to do so at this time. In order to move forward, the reasoning for the low peak middle must be understood and a

better fit function must be found.

VI. ACKNOWLEDGEMENTS

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