

# Delocalization of Quantum Particles in a Time-Dependent Potential

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## Abstract

The motion of an electron in a disordered time-dependent potential is an area of interest in theoretical condensed matter physics. Random “white noise” time-dependent potentials are well studied [1]. We investigate a sinusoidal “Floquet” time-dependent potential, specifically, the localization of one-dimensional Floquet lattice systems in comparison to white noise time-dependent lattice systems. Electrons in a time-independent potential in one and two dimensions are by nature localized [2]. In three dimensions, electrons are localized below the “mobility edge” [3]. Both Floquet and white noise systems delocalize as time progresses. If the coefficient of dynamic onsite energy is greater than the coefficient of static onsite energy, the average state of the white noise system is more extended than the average state of the Floquet system. If there is no static onsite energy, the white noise system is more extended than the Floquet system. If the coefficient of static onsite energy is nonzero, the Floquet system is most extended when the coefficient of static onsite energy is equal to the coefficient of dynamic onsite energy.

## 1. Introduction

In solid state quantum mechanics, Bloch’s theorem yields the solution to Schrodinger’s equation for a particle subject to a spatially periodic potential. An example of this system is electrons in a lattice of equally spaced nuclei. The resulting wave functions of these electrons are spread out over the lattice, thus the electrons conduct electricity [4].

Anderson introduced time-independent disorder into a spatially periodic potential. In a one-dimensional and two-dimensional lattice, the resulting wave functions are contained to regions of the lattice regardless of the extent of randomness [2]. This phenomenon is known as localization. In a material, localization corresponds to insulating properties.

Interest then developed in the introduction of time-dependent disorder and time-independent disorder into a spatially periodic potential. Numerical studies discuss the consequences of a dynamic disorder that has no memory of the potential at an earlier time. In this potential, small amounts of dynamic disorder trigger delocalization of electrons as neighboring orbitals are brought into resonance. Large amounts of dynamic disorder localize the electrons [1].

Recent questions concern a periodic time-dependent potential. The disorder of this dynamic potential oscillates as time progresses, thus the dynamic disorder does have memory of its earlier state. The investigation of this potential was the focus of my summer research.

## 2. Background

### 2.1 Introduction to Schrodinger's Equation

The position of a single classical particle in space is localized at a point. By nature, it has an exact location that can be determined by measurement. However, the exact position of the particle does not yield any information on the evolution of position with time. The classical approach using Newtonian mechanics uses net force, initial position, and initial velocity to solve for the position at any given time. This method is appropriate at the macroscopic level and validated by experimental results. However, when working with atomic and subatomic particles, there are discrepancies between the classical models and the experimental results. These anomalies are explained by quantum theory [5]. Quantum mechanics takes a statistical approach by modeling the particle as a wave function  $\Psi(x, t)$ . It does not provide the exact location of where the particle is, only where it is likely to be [4]. The probability of being at position  $x$  at time  $t$  is given by  $|\Psi(x, t)|^2$ .

From conservation of energy, the total energy of the system must equal the potential energy plus the kinetic energy. Schrodinger's equation is an analogous statement, though it takes a different form. For simplicity, assume there is a free particle moving in one dimensional space subjected to the time-independent potential  $V(x)$ . The corresponding Schrodinger's equation is as follows:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + V(x)\Psi(x, t) = i\hbar \frac{\partial \Psi(x, t)}{\partial t}$$

Then dividing the entire equation by the wave function yields:

$$-\frac{\hbar^2}{2m\Psi(x, t)} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + V(x) = \frac{i\hbar}{\Psi(x, t)} \frac{\partial \Psi(x, t)}{\partial t}$$

Next, assume the wave function is separable such that  $\Psi(x, t) = \phi(t)\psi(x)$ . Applying these conditions turns the partial differential equation into an ordinary differential equation. With minor simplifications, this yields:

$$-\frac{\hbar^2}{2m\psi(x)} \frac{d^2\psi(x)}{dx^2} + V(x) = \frac{i\hbar}{\phi(t)} \frac{d\phi(t)}{dt}$$

Spatial components are equivalent to the temporal components, so it must be the case that this equation is equal to a constant. Otherwise, varying time would affect the position-dependent functions and varying position would affect the time-dependent functions. Since position and time are not coupled, that cannot be the case. The components of the spatial equation represent kinetic energy and potential energy. Therefore, the constant must be the total energy of the system.

$$-\frac{\hbar^2}{2m\psi(x)} \frac{d^2\psi(x)}{dx^2} + V(x) = \frac{i\hbar}{\phi(t)} \frac{d\phi(t)}{dt} = E$$

## 2.2 Linear Algebra in Quantum Mechanics

Above, Schrodinger's equation was introduced in differential form. However, it is often more convenient to represent Schrodinger's equation using operators and bra-ket notation. This requires some understanding of linear algebra.

Vectors can be represented using bra-ket notation. In this notation, a vector  $x$  is denoted  $|x\rangle$ . The conjugate of a vector  $y$ , typically represented  $y^*$ , is denoted  $\langle y|$ . The inner product is then symbolized:  $\langle y|x\rangle$ . For a discrete vector:

$$\langle x|y\rangle = \vec{x}^* \cdot \vec{y}$$

A function is simply a continuous vector. The inner product for a function is then:

$$\langle f(x)|g(x)\rangle = \int f(x)^* g(x) dx$$

A vector operator is a matrix that is a linear mapping from one vector space to another vector space. Assume there is an operator  $\hat{O}$  that maps vector space  $V$  to vector space  $W$ . There may exist a vector  $|v\rangle$  in  $V$  such that when  $\hat{O}$  operators on  $|v\rangle$  the result is a scalar multiple of itself. These special vectors are known as eigenvectors, and these scalar values are known as eigenvalues:

$$\hat{O}|v\rangle = \lambda|v\rangle : \lambda \text{ is a scalar}$$

The idea of vector operators translates directly to function operators. Function operators map one function space to another function space. There may exist functions in the function space such that the function operator  $\hat{T}$  operating on a function results in a scalar multiple of itself. These resulting scalars are eigenvalues, and their corresponding functions are eigenfunctions.

$$\hat{T}|f(x)\rangle = \alpha|f(x)\rangle : \alpha \text{ is a scalar}$$

There is a subset of operators known as Hermitian operators. All observables in quantum mechanics are associated with Hermitian operators [4]. If an operator is Hermitian, it has three implications:

1. The eigenvalues are real
2. The eigenstates are orthonormal
3. The eigenstates form a complete basis

It then follows that the sum of the outer products of all eigenstates of a Hermitian operator is the identity operator [4]. If  $\{|f_n\rangle\}$  is an orthonormal discrete basis, this is represented:

$$1 = \sum_n |f_n\rangle\langle f_n|$$

If  $\{|f_n\rangle\}$  is an orthonormal continuous basis:

$$1 = \int |f_n\rangle\langle f_n| dn$$

Both vector operators and function operators are considered Hermitian if the conjugate transpose of the operator acting on the conjugate member of the inner product is equal to the operator acting on the original function or vector:

$$\langle \hat{O}^\dagger x | y \rangle = \langle x | \hat{O} y \rangle \Leftrightarrow \hat{O} \text{ is Hermitian}$$

$$\langle \hat{T}^\dagger f(x) | g(x) \rangle = \langle f(x) | \hat{T} g(x) \rangle \Leftrightarrow \hat{T} \text{ is Hermitian}$$

For a function operator, the inner product above is represented by an equation of two integrals:

$$\int \hat{T}^\dagger f(x)^* g(x) dx = \int f(x) \hat{T} g(x) dx$$

A vector operator is simply a matrix. Therefore, checking if the operator is Hermitian can be simplified. A vector operator is Hermitian if the conjugate transpose of the matrix  $\hat{O}^\dagger$  is equal to the original matrix  $\hat{O}$  [4][6].

$$\hat{O}^\dagger = \hat{O} \Leftrightarrow \hat{O} \text{ is Hermitian}$$

### 2.3 Operators in Quantum Mechanics

Above, we introduced the idea that observables are represented by Hermitian operators. The Hamiltonian is the Hermitian operator where the eigenvalues are the possible energy measurements. Using the spatial Schrodinger's equation resulting from a potential applied to a free particle moving in one dimensional space we can define the Hamiltonian operator  $\hat{H}$  for this system:

$$\hat{H}|\psi_n(x)\rangle = \left[ -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right] |\psi_n(x)\rangle = E_n |\psi_n(x)\rangle$$

Since the Hamiltonian is a Hermitian operator, if  $|\psi_n(x)\rangle$  is an orthonormal eigenstate of the Hamiltonian:

$$1 = \int |\psi_n(x)\rangle\langle\psi_n(x)| dn$$

The operator for the temporal Schrodinger's equation is referred to as the time-evolution operator:

$$\hat{E}|\phi_n(t)\rangle = \left[ i\hbar \frac{d}{dt} \right] |\phi_n(t)\rangle = E_n |\phi_n(t)\rangle$$

It too has eigenvalues of energy, and it can be used to describe how the eigenstates evolve with time. A solution to the temporal Schrodinger's equation for any system is:

$$\phi_n(t) = c_n e^{-\frac{iE_n t}{\hbar}} : c_n \text{ is a constant}$$

An important consequence of assuming the wave function is separable is that all states are stationary states [4]. While the wave function is time-dependent, the probability density of the state does not depend on time:

$$|\Psi(x, t)|^2 = \Psi(x, t)\Psi^*(x, t) = \phi(t)\psi(x)\phi^*(t)\psi^*(x) = |\psi(x)|^2$$

The time evolution operator can then be defined:

$$\hat{E} = \int e^{-\frac{iE_n t}{\hbar}} |\psi_n(x)\rangle\langle\psi_n(x)| dn$$

The equivalent statement for a discrete orthonormal basis  $\{|\psi_n\rangle\}$  is:

$$\hat{E} = \sum_n e^{-\frac{iE_n t}{\hbar}} |\psi_n\rangle\langle\psi_n|$$

This allows us to construct the time-evolution operator using quantities from the spatial state [8].

### 3. Model

#### 3.1 Localization and Inverse Participation Ratio

There are several forms of insulators. In the simplest “band insulators” the energy levels are entirely filled, and there is energy gap separating filled levels from unfilled levels. In conductive materials, localization is a form of phase transition that causes the material to express insulating properties as electrons become “stuck” at points on the lattice and classical diffusion ceases to exist [2].

The inverse participation ratio (IPR) allows us to qualitatively measure the number of states a particle is extended over. The IPR yields a numeric value, though this quantity is more of a relative classification than an absolute classification. Meaning, an IPR of a single state holds little to no significance by itself. However, by comparing IPR values between states we can conclude which state is more localized. For a discrete normalized state  $v$  the IPR is defined:

$$IPR = \frac{1}{\sum_i (|v_i|)^4}$$

Localization can be demonstrated in the normal modes of a classical mass spring system. The normal modes are frequencies at which the system will oscillate forever in the absence of driving or damping forces [7]. A system of  $N$  springs and masses are connected in a circular series. Let there be  $N-1$  identical springs of spring constant  $k$  and a single unique spring with a spring constant  $k'$  such that  $k' > k$  (Figure 1).

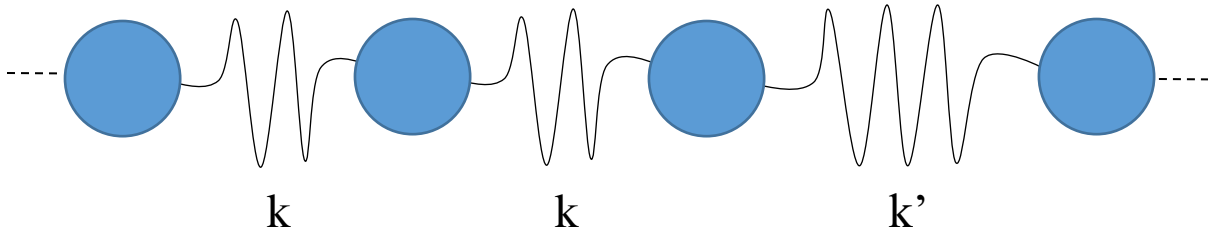


Figure 1: A circular chain of  $N$  identical masses,  $N-1$  identical springs (spring constant  $k$ ), and a single unique spring (spring constant  $k'$ ).

A circular chain of all identical masses and springs is a linearly dependent system. The degenerate eigenvalues are illustrated by pairs of normal modes with equivalent energies (Figure 2). As energy is added to the system, the corresponding natural frequencies follow a trend. If there is a unique spring in a circular chain of otherwise identical masses and springs, the normal modes demonstrate interesting characteristics (Figure 3). The energies of the normal modes are no longer degenerate, and the normal mode corresponding to the greatest oscillation of the unique spring is the highest energy mode.

The IPR of a state in a mass spring system represents the number of springs “participating” in the state. In a system of all identical masses and springs, all the modes are extended over many of the springs (Figure 4). In the system containing a unique spring, the highest energy mode is localized (Figure 5). Only the unique spring and nearby springs are participating in this mode.

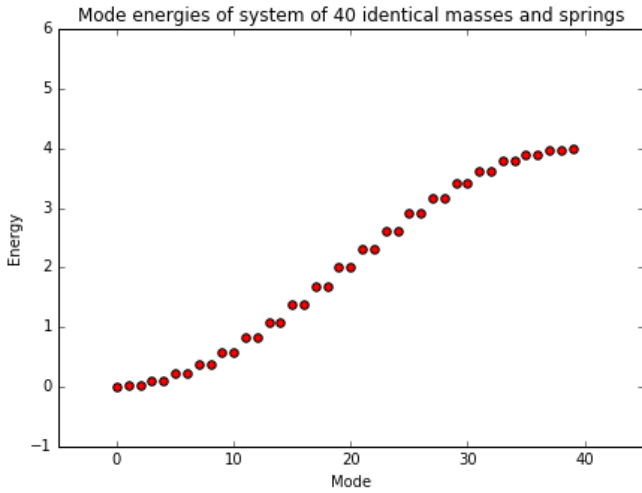


Figure 2: The modes with the maximum and minimum energies are the only nondegenerate modes for a circular chain of identical masses and springs.

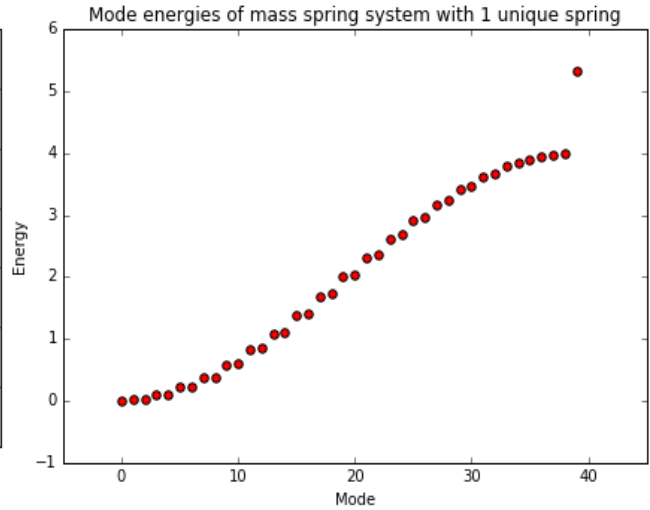


Figure 3: Increasing the spring constant of a single spring in a system of otherwise identical masses and springs results in unique energies for every mode and a significant input of energy is required to activate the highest energy mode. In this case  $k' = 2k$ .

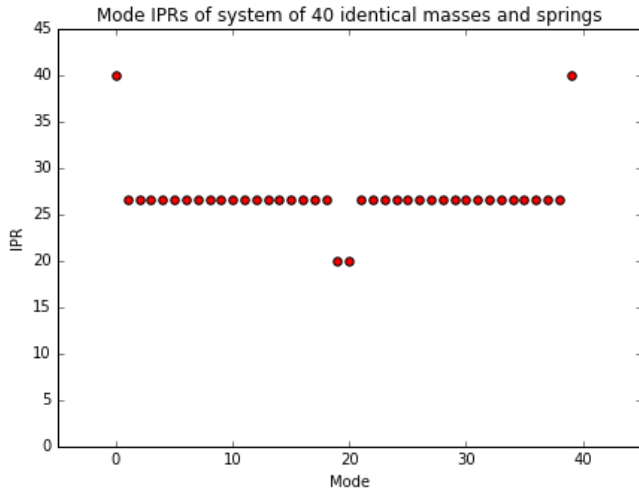


Figure 4: Even in a system of all identical masses and springs, the IPR values are not all equal. The lowest and the highest energy modes are the most extended states.

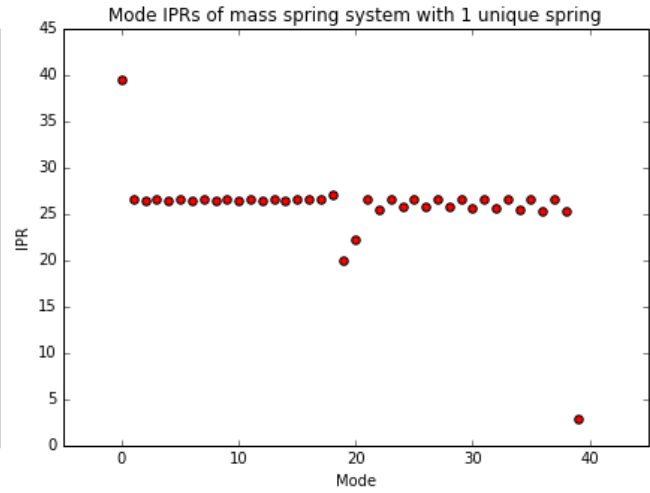


Figure 5: Replacing one of the identical springs with a unique spring creates disorder and causes localization in the highest energy mode. This is identified by the relatively low IPR value. In this case  $k' = 2k$ .

The motion of electrons in a disordered time-independent potential demonstrate an analogous behavior to the mass-spring system. These systems undergo “Anderson Localization” as particles may become “stuck” in low potential sites. However, if the potential becomes time-dependent, the material may regain its conductive properties. This transition is known as delocalization (Figure 6).

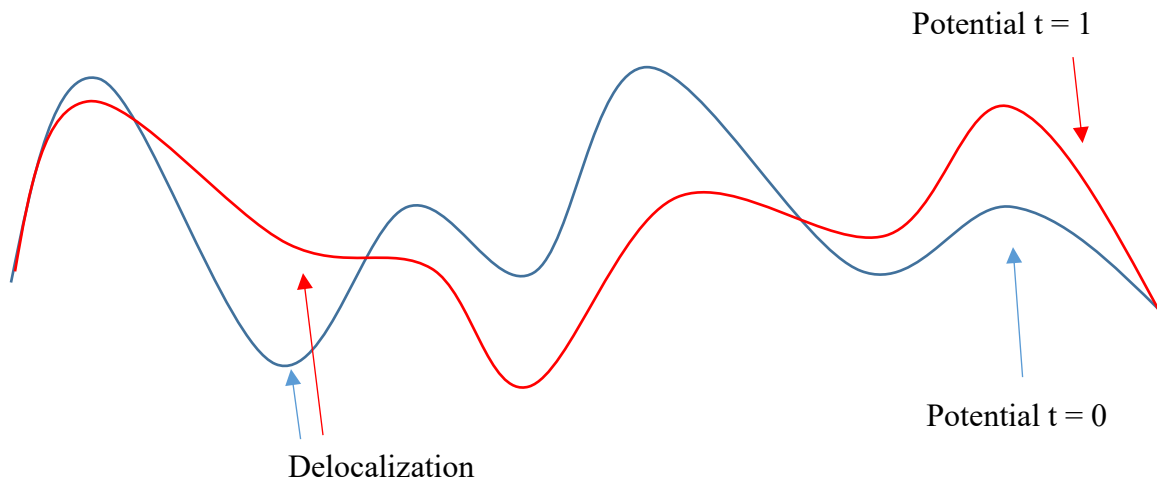


Figure 6: Delocalization occurring in a time-dependent potential. Particles that are localized to a low potential trough will delocalize as the potential increases at a later time and neighboring sites provide lower energy states.

### 3.2 Onsite Energy

We studied a lattice model of quantum motion. Every orbital in a lattice has a time-dependent onsite energy that can be thought of as an “activation energy”. That is, an orbital’s onsite energy is the minimum energy a particle needs to bind to the corresponding orbital, excluding contributions from external interactions. Every orbital has four time-independent attributes that contribute to determining the onsite energy: an offset  $d$ , a phase  $\phi$ , an amplitude  $c$ , and a period  $T$ . Both the phase and the offset are unique to each orbital, uniformly distributed random, set at system initialization, and remain a constant. The period and amplitude of oscillation are attributes of the lattice inherited by individual orbitals, meaning these values are shared by all orbitals in the lattice.

$$\epsilon_i(t) = d_i + c \cos\left(\frac{2\pi t}{T} + \phi_i\right)$$

In order to use the onsite energy in the Hamiltonian, the Hamiltonian must be time-independent. To accommodate this restriction, the onsite energy is redefined with quasi time-dependence. The period  $T$  is partitioned into  $M$  steps, and at each step  $m$  the onsite energy is constant. This approach transforms the onsite energy into a discrete function of  $m$ :

$$\epsilon_i(m) = d_i + c \cos\left(\frac{2\pi m}{M} + \phi_i\right) : m \in \mathbb{N}$$

This manipulation only makes the onsite energy a piecewise constant function. It does not eliminate dependence on time. To better illustrate this, the function can be rewritten in terms of time  $t$ :

$$\epsilon_i(t) = d_i + c \cos\left(\frac{2\pi}{M} \left\lfloor \frac{t}{M} \right\rfloor + \phi_i\right)$$

While the result of the floor division is constant, the onsite energy is effectively time-independent. Therefore, we can construct a Hamiltonian for these increments of time.

### 3.3 Hamiltonian

The wave function for a system of orbitals aligned in one dimension is represented as a column vector. Each row corresponds to the particle occupation of a specific orbital. In a classical system, the particle is fully confined to a single orbital at any given time. However, for a quantum system the wave function of the particle can be extended over many different states. Thus, the wave function for the quantum system is a superposition of the classical states. The basis vectors for the quantum state in this system include all possible combinations of states where each particle is confined to a single orbital. The basis vectors can then be represented as a binary number in bra-ket notation where zeroes represent unoccupied orbitals, and ones represent occupied orbitals. In the event of multiple particles introduced into a lattice of orbitals, the particles are considered indistinguishable.



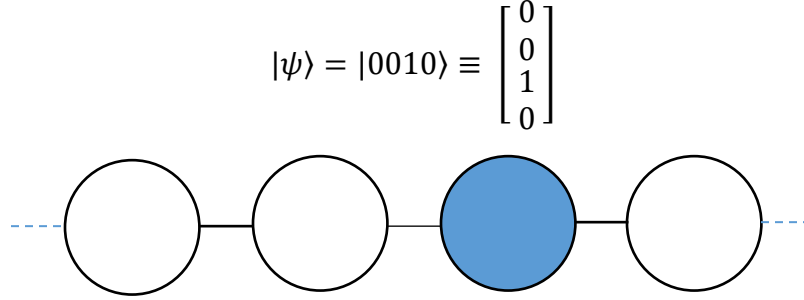


Figure 7: For a system of 4 orbitals aligned in one dimension containing 1 particle, the basis vector above represents the state in which the particle is confined to the 3<sup>rd</sup> orbital.

The Hamiltonian for a single particle in a one-dimensional lattice consists of the onsite energy term and tunneling energy term. The concept of particle tunneling is unique to an ensemble of orbitals. A particle in the system can occupy individual orbitals and tunnel to neighboring orbitals. The energy required for a particle to tunnel is time-independent and a global property of the system. In other words, this energy does not depend on the specific orbitals traversed.

In order to rigorously explain onsite energy and tunneling energy, we use the creation operator  $\hat{a}^+$  and destruction operator  $\hat{a}$ . A creation operator acting on an empty orbital  $i$  creates a particle in the  $i$ th orbital. Similarly, a destruction operator acting on an occupied orbital  $i$  destroys a particle in the  $i$ th orbital. Because of the Pauli principle, it is not possible to add a particle to an occupied orbital or destroy a particle in an empty orbital, therefore attempting to do so has no resulting wave function. In order to conserve particle number, creation operator and destruction operators must come in pairs.

- (1)  $\hat{a}^+ |0\rangle = |1\rangle$
- (2)  $\hat{a} |1\rangle = |0\rangle$
- (3)  $\hat{a}^+ |1\rangle = 0$
- (4)  $\hat{a} |0\rangle = 0$

The contribution of energy due to onsite energy, is equal to the summation of the onsite energy of all occupied orbitals. To determine which orbitals are occupied, a particle is destroyed then created in each orbital. If there is no resulting wave function, the orbital was empty. If there exists a resulting wave function, it is necessarily identical to the initial wave function and contributes a non-trivial onsite energy to the Hamiltonian.

$$\hat{H}_{onsite} = \sum_i \epsilon_i(t) \hat{a}_i^+ \hat{a}_i$$

The tunneling energy contribution to the Hamiltonian is the global tunneling constant multiplied by the set of all possible wave functions that result from a particle tunneling to an unoccupied adjacent orbital. Therefore, the creation operator and destruction operator must operate on consecutive sites in each lattice dimension. Meaning, adding a particle to site  $i$  in a lattice must be accompanied by destroying a particle in the  $i+1$  or  $i-1$  site. In order for our small lattices to act similar to a large lattice, particles are allowed to tunnel via periodic boundaries.

$$\hat{H}_{tunneling} = -\tau \sum_i (\hat{a}_i^+ \hat{a}_{i+1} + \hat{a}_{i+1}^+ \hat{a}_i)$$

We can then define the one-dimensional Hamiltonian for a lattice system with a single occupation from the sum of onsite energy and the tunneling energy contributions.

$$\hat{H} = -\tau \sum_i (\hat{a}_i^+ \hat{a}_{i+1} + \hat{a}_{i+1}^+ \hat{a}_i) + \sum_i \epsilon_i(t) \hat{a}_i^+ \hat{a}_i$$

For a single particle introduced into an  $N$ -site one-dimensional lattice, the resulting Hamiltonian operator is an  $N$  by  $N$  matrix with non-zero diagonal, off-diagonal, and far anti-diagonal terms (Figure 8). The diagonal terms correspond to each orbital's onsite energy. The off diagonals and far anti-diagonals correspond to the tunneling energy. The far anti-diagonal terms arise from periodic boundaries.

$$\hat{H} = \begin{bmatrix} 0.2 & 1 & 0 & 0 & 0 & 1 \\ 1 & -0.5 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0.1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.9 & 1 & 0 \\ 0 & 0 & 0 & 1 & -0.1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0.3 \end{bmatrix}$$

Figure 8: Example matrix for a system of a single particle introduced into a 6-site one-dimensional lattice. In this example  $\tau = -1$  and  $\epsilon_i \in [-1,1]$ .

Upon introducing multiple particles into a lattice system, an additional energy must be taken into account: interaction energy between particles. This contributes a static value  $V$  for every adjacent pair of orbitals if and only if adjacent orbitals are occupied.

$$\hat{H} = -\tau \sum_i (\hat{a}_i^+ \hat{a}_{i+1} + \hat{a}_{i+1}^+ \hat{a}_i) + \sum_i \epsilon_i(t) \hat{a}_i^+ \hat{a}_i + V \sum_i \hat{n}_i \hat{n}_{i+1}$$

In the case of multiple particles introduced into an  $N$ -site one-dimensional lattice, the resulting Hamiltonian is an  $2^N$  by  $2^N$  block diagonal matrix (Figure 9). Each block is a square matrix of size  $N$  choose  $r$ , where  $N$  is the total number of lattice sites and  $r$  is the number of particles in the lattice. The uppermost left block of the block diagonal matrix represents an empty lattice. Moving towards the bottommost right, each block traversed represents the addition of a particle into the lattice. In the bottommost right block,  $N$  particles occupy an  $N$ -site lattice.

[ 0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	]
[ 0.	-0.79	-1.	0.	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	]
[ 0.	-1.	-0.178	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	]
[ 0.	0.	-1.	-0.719	-1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	]
[ 0.	-1.	0.	-1.	0.867	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	]
[ 0.	0.	0.	0.	0.	-1.968	-1.	0.	0.	-1.	0.	0.	0.	0.	0.	0.	]
[ 0.	0.	0.	0.	0.	-1.	-1.508	-1.	-1.	0.	-1.	0.	0.	0.	0.	0.	]
[ 0.	0.	0.	0.	0.	0.	-1.	-1.897	0.	-1.	0.	0.	0.	0.	0.	0.	]
[ 0.	0.	0.	0.	0.	0.	-1.	0.	-0.922	-1.	0.	0.	0.	0.	0.	0.	]
[ 0.	0.	0.	0.	0.	-1.	0.	-1.	-1.	0.689	-1.	0.	0.	0.	0.	0.	]
[ 0.	0.	0.	0.	0.	0.	-1.	0.	0.	-1.	-0.851	0.	0.	0.	0.	0.	]
[ 0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-3.686	-1.	0.	-1.	0.	]
[ 0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	-2.1	-1.	0.	0.	]
[ 0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	-2.641	-1.	0.	]
[ 0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-1.	0.	-1.	-2.029	0.	]
[ 0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	-4.819	]

Figure 9: Example of the block diagonal matrix that arises when introducing multiple particles into a 4-site one-dimensional lattice.

### 3.4 Floquet Operator

When the Hamiltonian is time-independent, the time-evolution operator describes how the wave function evolves with time. However, our Hamiltonian is periodically time-dependent. Therefore, in order to access information on how the system evolves with time, we rely on the Floquet operator. Recall the time-evolution operator can be defined in terms of the eigenstates and eigenvalues of the Hamiltonian:

$$\hat{E} = \sum_n e^{-\frac{iE_n t}{\hbar}} |\psi_n\rangle\langle\psi_n|$$

However, in order for this to be true, the Hamiltonian must be time-independent. Our Hamiltonian has a periodic time-dependent onsite energy. The onsite energy has a period of  $T$  that is partitioned into  $M$  steps. Therefore, the length of each time step is  $T/M$ . When the onsite energy is time-independent, the wave function is separable and the time-evolution definition above is valid. For each time step:

$$\hat{E}_m = \sum_n e^{-\frac{iE_n T}{\hbar M}} |\psi_n\rangle\langle\psi_n|$$

The Floquet operator is our mechanism for judging the relative localization or extension of the basis states of the system. For one period as described above, the Floquet operator is defined:

$$\hat{U} = \hat{E}_{M-1}\hat{E}_{M-2} \dots \hat{E}_2\hat{E}_1\hat{E}_0$$

For a singly occupied lattice, the IPR is calculated from the eigenvectors of the Floquet operator. In the event that multiple particles are introduced into a lattice, the IPR is calculated from the eigenvectors of the reduced density matrices of the Floquet operator.

### 3.5 White Noise versus Floquet Time-Dependence

In lattice systems with random or “white noise” time-dependence, each lattice site has a uniformly distributed random static onsite energy  $d$  and dynamic onsite energy  $\lambda$ . The dynamic onsite energy term is rerandomized at every time step. These systems are well studied and provide a benchmark for our comparisons. The resulting onsite energies for the white noise system are:

$$\epsilon_i(m) = d_i + c\lambda_i : \lambda_i \in [-1, 1]$$

Recall the onsite energy in the Floquet system:

$$\epsilon_i(m) = d_i + c \cos\left(\frac{2\pi m}{M} + \phi_i\right)$$

In both onsite energy equations,  $d$  is the coefficient of static onsite energy and  $c$  is the coefficient of dynamic onsite energy. Both onsite energies are implicitly dependent on  $t$ . The period  $T$  is divided into  $M$  partition. During each partition  $m$ , the onsite energy is effectively static.

## 4. Results

### 4.1 Localization of Lattice Energy

We begin by observing the energy of the states that result from introducing a single particle into a 300-site one-dimensional lattice system. Here, the IPR is calculated using the eigenvectors of the Hamiltonian. Similar to the mass spring system, the high and low energy states exhibit localization as characterized by a relatively low IPR value. Since the Hamiltonian is generated at each time step, this trend of localization at energy extrema does not depend on time. It is a characteristic of the one-dimensional lattice system.

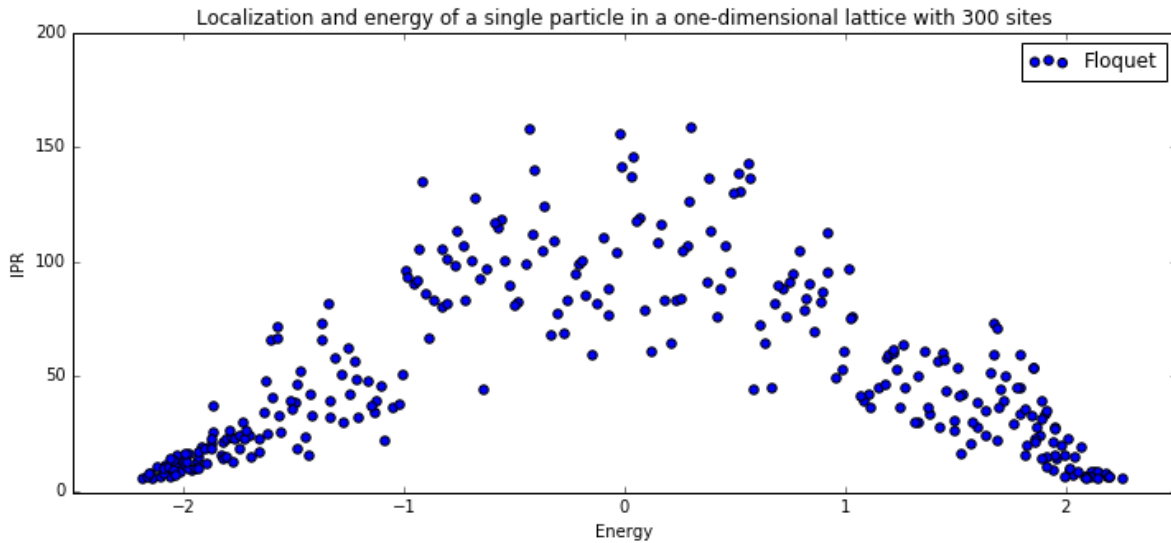


Figure 10: In a one-dimensional lattice localization occurs in the high and low energy states.

## 4.2 Varying Coefficient of Static Onsite Energy

For both white noise and Floquet systems, the static onsite energy for each lattice site is a uniformly distributed random number between  $[0, d]$  where  $d$  is the coefficient of static onsite energy. In this model, the coefficient of dynamic onsite energy is set to 1, meaning the dynamic onsite energy can be anywhere between  $[-1, 1]$ . Increasing  $d$  more than  $c$  decreases particle mobility and causes localization in both Floquet and white noise systems (Figure 11).

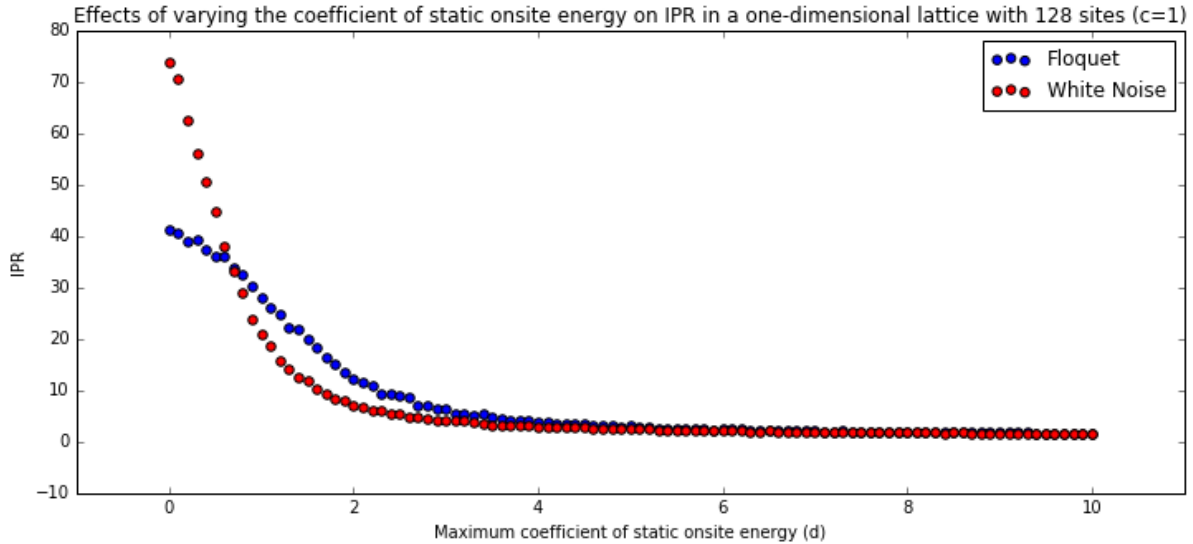


Figure 11: Comparison of varying the coefficient of static onsite energy in Floquet and white noise lattices. The two plots represent an average of 10 one-dimension lattices each having 128 sites,  $c = 1$ ,  $T = 6$ , and 100 time steps.

When the static onsite energy is less than the dynamic onsite energy ( $d < c$ ) the white noise system is more extended than the Floquet system. This implies if there is little to no variation in the static onsite energy, an oscillating dynamic onsite energy decreases particle mobility relative to the white noise system. Both systems are most extended when there is no variation of static onsite energy between lattice sites.

## 4.3 Varying Coefficient of Dynamic Onsite Energy

The coefficient of dynamic onsite energy allows the dynamic onsite energy to range between  $[-c, c]$  for both the white noise and Floquet systems. The Floquet model is more extended than the white noise model when the coefficient of dynamic onsite energy is less than approximately two times the coefficient of static onsite energy. However, the Floquet system begins to delocalize in this region (Figures 12, 13). The optimal parameters for extension for the Floquet model is around  $c = d$ . In a system with  $c = d$ , onsite energy fluctuations are more likely to cause an occupied orbital's energy to surpass that of its neighbor and trigger tunneling to a neighboring orbital.

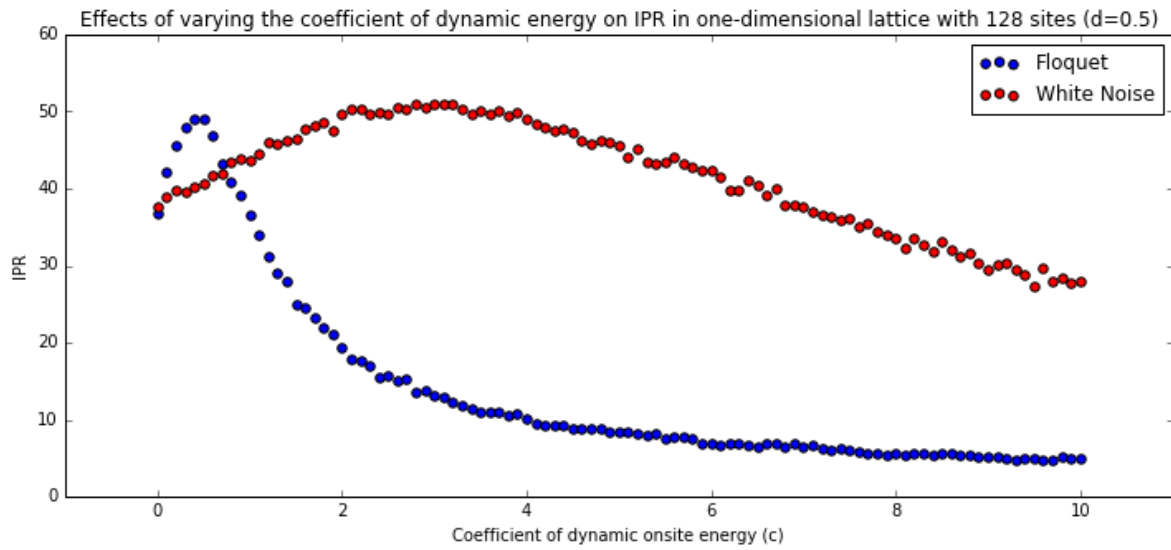


Figure 12: Comparison of varying the coefficient of dynamic onsite energy in Floquet and white noise lattices. The two plots represent an average of 10 one-dimension lattices each having 128 sites,  $d = 0.5$ ,  $T = 6$ , and 100 time steps.

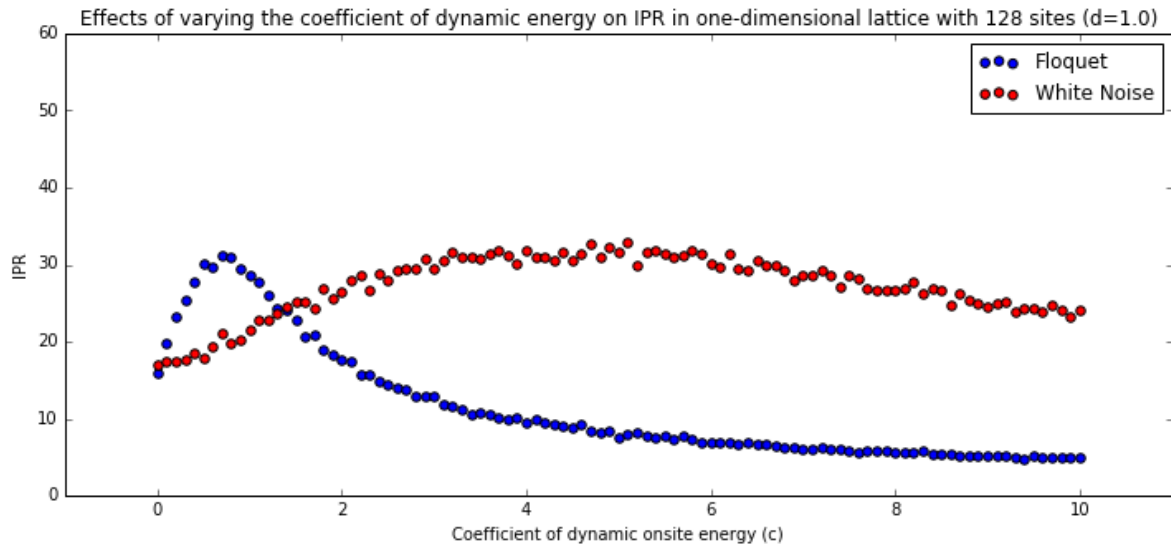


Figure 13: Comparison of varying the coefficient of dynamic onsite energy in Floquet and white noise lattices. The two plots represent an average of 10 one-dimension lattices each having 128 sites,  $d = 1.0$ ,  $T = 6$ , and 100 time steps.

The white noise system delocalizes at a much greater value of  $c$ . Due to the oscillatory nature of the Floquet system, phase differences between neighboring orbitals may decrease particle mobility. Even if an occupied orbital has more energy at its peak than the energy at the trough of a neighboring orbital, if these two orbitals are in phase, particle tunneling will never occur. In the white noise system, at each time step orbitals are randomly assigned a dynamic energy in the dynamic energy domain. Without oscillation and phases, if an occupied orbital and an unoccupied neighboring orbital have overlapping energy domains, particle tunneling is more likely to occur in the white noise system than in the Floquet system.

## 5. Conclusion

This model of the Floquet system was originally developed in Python, and later developed in C to improve performance. Unfortunately, converting to C consumed a large amount of time and limited the number of results attained.

The majority of time was spent researching a system with a single particle, however the multiple particle system (briefly explained in Section 3.3) is an area of interest for future work. With multiple particles, one must take into account the interaction between particles. In this model, particle mobility depends both on the energy of neighboring orbitals and whether or not the neighboring orbital is occupied. With multiple particles, there is the possibility of localization due to a clustering of the particles.

The model developed in this paper is restricted to a single dimension. All parameters equal, adding multiple lattice dimensions increases the number of neighboring orbitals and likely increases state extension. It would be interesting to see if multiple dimensions change the relationship between the Floquet and white noise systems. Also, there is the question of a dimension in which Floquet and white noise systems are indistinguishable. Although this may not have an obvious physical interpretation it may yield an upper bound for which the Floquet implementation is relevant and interesting.

Even for a single particle in a one-dimensional lattice, there still remain areas of interest. I never attained reliable information on how many samples  $M$  the period should be partitioned into. As  $M$  increases, the periodic time-dependence will better model continuous time-dependence, however this requires more computation time. It is unclear what values of  $M$  are ideal.

In this study, the results indicate when the coefficient of dynamic onsite energy is greater than the coefficient of static onsite energy, the average state of the white noise system is more extended than the average state of the Floquet system. If there is no static onsite energy, the white noise is more extended than the Floquet system. If the coefficient of static onsite energy is nonzero, the Floquet system is most extended when the coefficient of static onsite energy is equal to the coefficient of dynamic onsite energy.

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