Temperature Effects and Oscillations on Vortices in Superfluid Helium

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A new measurement technique for studying kelvin wave oscillations along vortices in superfluid helium was tested. In the process of testing this technique, a dependence on temperature was discovered in addition to strange periodic behavior that may not be fully described by this temperature dependence. This new-found dependence is discussed as a new measurement tool for studying vortices. A need for a better understanding of pinning on a lip that separates two different cell diameters is also discussed.

Keywords: Kelvin waves, superfluid helium vortices

INTRODUCTION

Vorticity in Superfluid Helium

Vortices are a very common phenomenon with applications to many different branches of science, as well as everyday life. An important example of vortex research is in type II superconductors. When subjected to a magnetic field, these superconductors will form magnetic vortices that penetrate the superconductor. If these vortices move, a non-zero resistance will result which is obviously an unwanted outcome. Preventing a vortex from moving is called "pinning" the vortex, and so pinning is important to understand. Pinning is also an interesting phenomenon when concerning neutron stars, which are thought to have superfluid cores. Pinning and de-pinning of vortices within this superfluid core are thought to be the reason for rotational glitches that occur in the neutron stars' rotational patterns.

Vorticity, ω is defined as the curl of a velocity field v in a fluid,

$$\omega = \nabla \times v \tag{1}$$

. A useful parameter, circulation, can be related to the vorticity by Stoke's theorem. Circulation is defined in the following way with its corresponding Stoke's theorem equivalent,

$$\kappa = \oint_{loop} v \cdot dl \tag{2}$$

$$\kappa = \iint_{surface} (\nabla \times v) dA, \tag{3}$$

the second form can now be related back to the vorticity definition, and we see that circulation is given by the surface integral of vorticity.

$$\kappa = \iint_{surface} \omega dA \tag{4}$$

Using a superfluid as the medium for measurement allows for such simplifications as a constant vortex core size [2], in addition to the ability to study just one vortex at a time. These simplifications are a result of a macroscopic wavefunction that describes the whole system of He-4 atoms. Very low temperatures cause the individual wavefunctions of the atoms to overlap and combine into a single-particle wavefunction. This transition occurs at a higher temperature for He-4 than He-3 because He-3 atoms must form pairs in order to behave bosonically. He-4 atoms are already bosons, so their wavefunctions do not need to remain distinct as they overlap. This leads to He-3 having a much lower superfluid transition temperature than He-4. Once the system can be described by a single wavefunction, a requirement that the wavefunction must be single-valued then requires quantization of the circulation in the helium. The phase of the superfluid, Θ is related to the velocity by

$$v = \frac{\hbar}{m_{He}} \nabla \Theta. \tag{5}$$

Plugging this expression into equation 2 will then yield the quantization of the circulation in the superfluid as seen by

$$\kappa = \frac{hn}{m_{He}}, \tag{6}$$

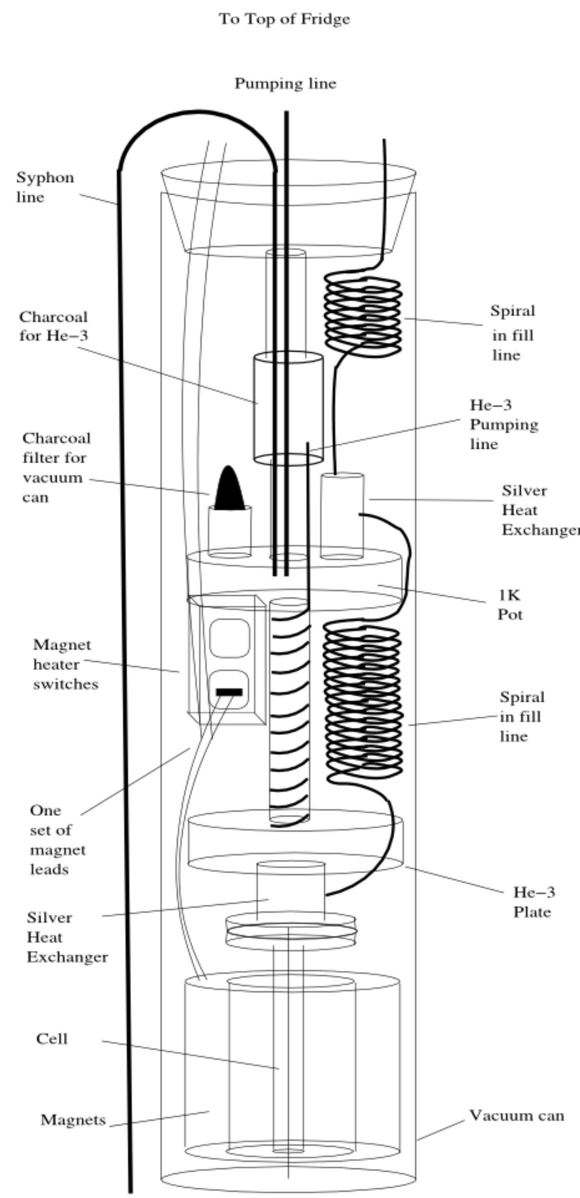
where h, n, and m_{He} are Planck's constant, some integer, and the mass of a helium atom, respectively. For there to be circulation in a superfluid, our loop must enclose the vortex core, which cannot be a superfluid. We know this because if we were to instead insert equation 5 into equation 3, we would then have $(\nabla \times (\nabla \Theta))$ under the integrand, which is zero! The vortex core creates a discontinuity so that circulation is allowed. We provide an energetically favorable core for the vortex in the form of a superconducting wire. The wire is favorable because the system will not have to spend any energy by converting some of the superfluid to normal in order to meet this non-superfluid core requirement. [1]

Trapping Circulation

The superfluid transition temperature for helium-4 is 2 kelvin, and we measure the system at around 315 millikelvin. To achieve these temperatures, we use a sorbtion-pumped helium-3 fridge, as seen in Figure 1. The first step of cooling is to place the fridge in a dewar of liquid helium-4. This will bring the system down to around 4 kelvin. Next, helium-4 is siphoned into a pot on the fridge, and pump on it with a roughing pump (that works really beautifully). This causes a process called evaporative cooling to take place, and will cool the plate to around 1.5 K. Though this is low enough for superfluid helium to be present, the signal to noise ratio is better at lower temperatures, so we'd really like to keep cooling. To get below 1.5 K, we now use He-3 (in a closed system), which has a lower boiling point and is thus easier to pump on. The closed system for the He-3 consists of a charcoal pump, or sorb. When the charcoal is cooled, the He-3 molecules are likely to stick to it, and the charcoal can then act as a pump for the He-3 for another round of evaporative cooling that will take the system down to our desired temperature for measurements. [1]

To actually trap vortices within the cell, we rotate the cryostat. Though it seems counterintuitive that a superfluid (which is characterized by zero viscosity) could interact with the wall to produce vortices, there is actually a well-measured critical velocity above which the superfluid is able to interact with the wall of the cell. [3] This velocity is easily reached by rotating the cryostat, and all that's left is to wait for one of the vortices to attach to the wire, as opposed to annihilating itself on the wall.

FIG. 1: Diagram of sorbtion-pumped He-3 fridge



MEASUREMENT TECHNIQUES

As briefly mentioned earlier, there is a superconducting wire that spans the length of the cell which is filled with superfluid helium-4. This wire is the basis for our measurements of circulation. There are also magnets which surround the cell, creating a magnetic field perpendicular to the wire. When a current is pulsed through the wire, it feels a Lorentz force which pulls it to the side much like the pluck of a guitar string. The vibration of the wire within the magnetic field creates a measurable emf for us to study. As the wire vibrates, it also begins to precess in the direction of the circulation on the wire, due to Bernoulli's law. The wire will precess moving perpendicular then parallel to the magnetic field and on as the vibration dissipates. This creates an envelope shape shown in Figure 2. The envelope is fit to the following expression,

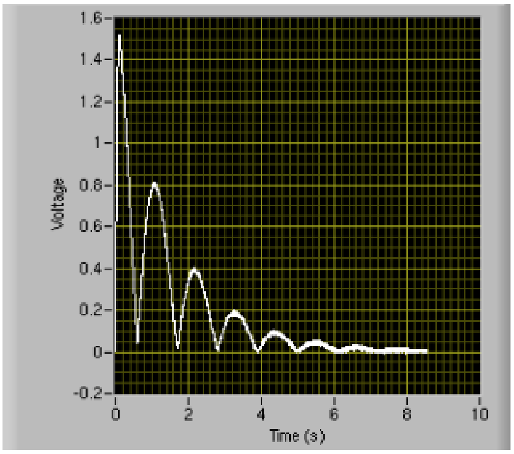
$$|Ae^{-\lambda t}\cos(d\omega t + \phi)|, \tag{7}$$

an exponential decay envelope with some amplitude A and phase shift ϕ. The $d\omega$ value within the cosine can be converted to a circulation value. These oscillations take 5-10 seconds to decay, meaning that we can extract a circulation value with this technique at that rate. This measurement technique is called the "pulsed mode."

The vortex phenomenon of interest in this paper is kelvin waves. A kelvin wave is a perturbation along a vortex. The lowest order of these waves can be seen when the vortex becomes pinned to its pin site. The end of the vortex attached to the wire acts as the first maximum and the end on the pin site acts as a node. As the wave oscillates along the vortex, the end of the vortex on the wire will move, changing the amount of circulation on the wire. This change of circulation can then be tracked by the pulsed mode every 5-10 seconds as previously mentioned. The lowest order kelvin waves have periods of around 42 seconds, [4] which is very reasonable to study at the 5-10 second rate of measurement. However, it is possible to excite higher order kelvin waves, which have much shorter periods, which makes the current sampling rate unreasonable. These waves exhibit nonlinear properties which can help to understand turbulence in superfluids, but a faster method of circulation measurement than the pulsed technique is needed to study them.

The new method of measurement is built off of measuring in fourier space. When a fourier transform of the

FIG. 2: Picture of the envelope signal that is used for the pulsed mode of measurement. Peaks correspond to when the vibrations of the wire are perpendicular to the magnetic field, and troughs are when the vibrations have processed to being parallel to the field.



envelope from equation 3 is done, the response is a double lorentzian shape with the form,

$$\frac{[a^3 + a(\omega - \omega_\circ)^2 + a^{(}d\omega)^2]^2 + [a^2(\omega - \omega_\circ) + (\omega - \omega_\circ)^3]^2}{[a^2 + (\omega - \omega_\circ + d\omega)^2][a^2 + (\omega - \omega_\circ - d\omega)^2]}, \quad (8)$$

where $d\omega$ is a measure of the separation of the two peaks of the double lorentzian and $\omega_\circ$ is the center point of the two peaks. [2] The frequency at each peak is a resonant mode of the wire. The separation of the peaks (again, represented by $d\omega$) is dependent on the amount of circulation on the wire, i.e. when there is more circulation, the peaks become more separated as shown in Figure 3. Because the separation of peaks represents circulation, it is possible to sample circulation just by watching the shape of the double lorentzian change. This is achieved by choosing a frequency that will yield the largest change with changing circulation (such as the slope next to a peak), and staying there to continuously take measurements. In the previous summer, this technique was verified but needed more testing.

RESULTS AND DISCUSSION

When tests were first being run on continuous mode, strange oscillations were seen in the wire signal. Two significant types of oscillations were seen, both seen without any effort to excite kelvin waves along the vortices. Oscillations started to be seen that were semi-periodic, but very strange in shape. It was then realized that these oscillations were matching up with small fluctuations in temperature from the temperature controller. The correlation can be seen in Figure 4. Tests were then run to ensure that the width of the double lorentzian (and therefore the corresponding circulation measurements) were unaffected by the temperature changes. There were now

FIG. 3: This figure shows the double lorentzian shape for three cases of circulation. N=0 is represented by the red line, N=1/2 is the blue line, and zero circulation is shown by the black line.

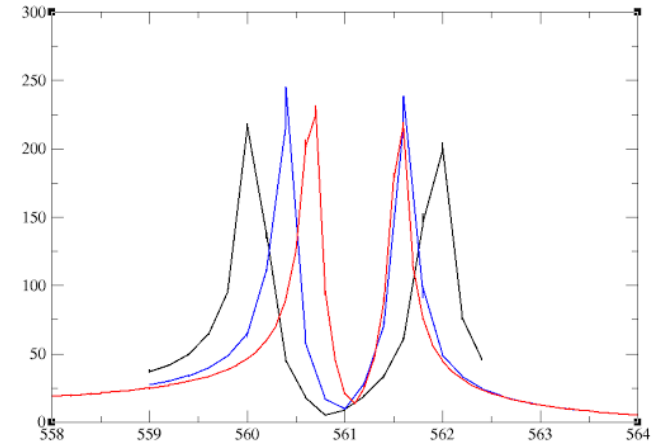
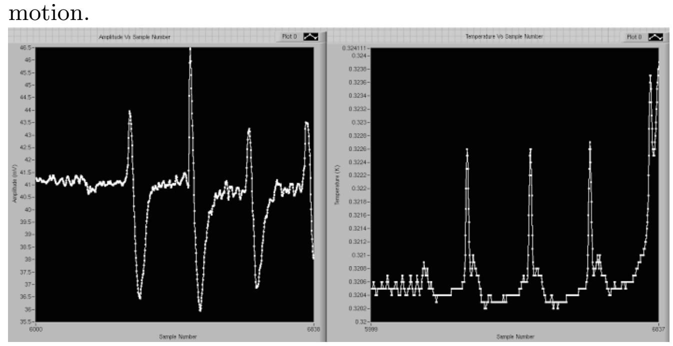


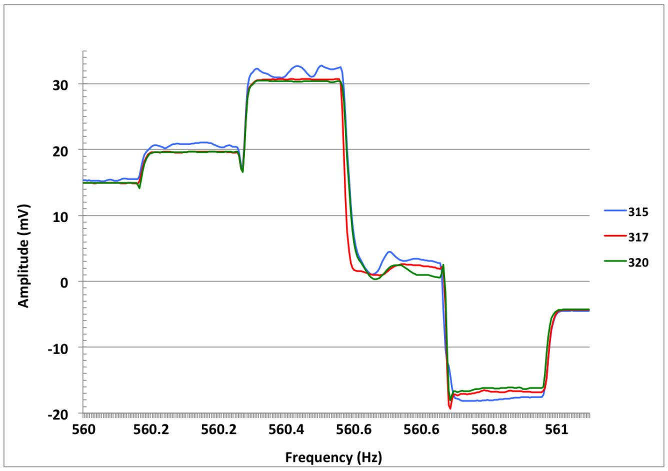
FIG. 4: The left graph shows the voltage response from the wire, and the right is showing the temperature fluctuations. Though it is obvious that the responses match the fluctuations, the response is still not perfectly correlated (as seen in the steep drop on the left side that has no matching cause on the right), suggesting the response is due to the vortex's motion.



two possibilities to be explored to learn more about what these temperature fluctuations were causing. The more boring possibility is that the change in temperature was simply yielding a different response from a singular frequency. The more interesting possibility would be that the temperature difference was causing a response from the vortex.

To test whether the temperature fluctuations were simply causing a change in frequency response, three identical sweeps were done at three different temperatures. The temperatures that we tested were 315, 317, and 320 mK. These differences in temperature are much larger than the fluctuations we were seeing in Figure 4. As seen in Figure 5, the levels at each frequency step are very nearly identical, meaning we were not getting a simple shift in frequency response. The phenomena observed from the temperature fluctuations are thus very likely to be some sort of response of the vortex as it interacts with a temperature gradient that is set up by these fluctua-

FIG. 5: Each of these sweeps were run on the same frequencies, to ensure that the frequency response was not the reason for the oscillations in the wire signals. That the levels are so close suggests the response is a vortex phenomenon, not just a shift in frequency response. It is worth noting that the lowest temperature case (315K) has considerably more oscillation on each level than the other two. This is likely because at lower temperatures there is lower dissipation, causing any vibration to persist longer at this temperature.



tions. The exact dynamics of the situation need to be studied in more detail. One complication to considering the vortex dynamics for this specific case could involve the boundary conditions of the test-cell being used over the summer, which will be discussed in more detail later.

A different and much rarer oscillation was also seen during measurement, which had a period of around 11 seconds, and can be seen in Figure 6. These oscillations were much clearer and more periodic than the oscillations that were earlier shown to mimic the temperature controller. The periods of these oscillations were far from the 42-second period of the lowest order kelvin wave oscillation, which we would have expected to see. This effect was first seen before the realization that temperature was having a definite effect on the vortices however, so it was later decided that an additional digit needed to be recorded in order to see the effects in the data. Since this temperature effect was not yet known, another possibility was considered that is still of importance. The particular test-cell being used at the time did not have a singular reliable pin site (i.e. a macroscopic bump on the wall of the cell), but instead had two different diameters with a lip that the vortex tended to pin to. Pinning on a lip is not as fully understood as pinning on a bump, so it is possible that in addition to the typical kelvin wave oscillations that occur vertically along the vortex, horizontal oscillations may also come about in this system, as pictured in Figure 7. If in fact these kelvin waves were being excited horizontally, these waves would have different boundary conditions than the vertical waves do, as the horizontal waves will be allowed to move on the lip rather than being pinned on that end. Further investigation would then be required because typically the pin-site is treated as a node for the oscillation, which is still true in the vertical direction, but needs to be reconsidered in the horizontal direction. This new possible direction of movement could also be making the temperature fluctuations mentioned earlier harder to interpret than if we could observe them while the vortex was more reliably pinned.

FIG. 6: These are the first oscillations seen when the continuous mode was run on a single frequency. The oscillation period is around 11s, which is much smaller than the period that would have been expected of the lowest order kelvin wave oscillations. Similar oscillations were seen later on with slightly different periods, but their cause is still unclear, and their occurrence was too rare to be studied in detail.

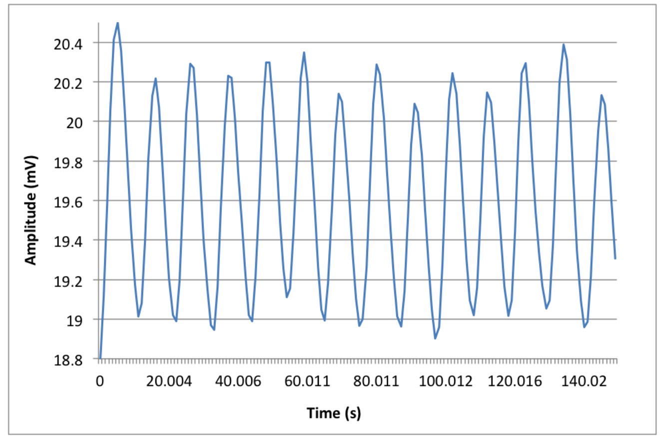
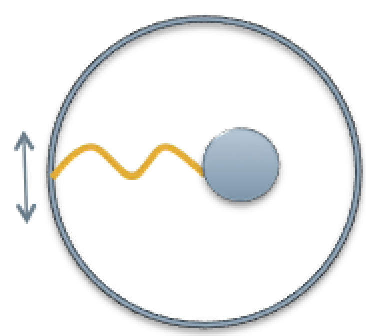


FIG. 7: Picture representing possible horizontal excitation of kelvin waves. In this hypothetical scenario, the wave would cause the vortex to oscillate along the lip of the cell. This wave would necessarily have a different period than that of the typical kelvin wave, which would cause the vortex to oscillate vertically with the pin-site acting as a node. Without a definite node at the pin-site it is unknown exactly how the vortex should behave.



CONCLUSIONS AND FURTHER QUESTIONS

Though the temperature dependence is still not wholly understood, it seems very apparent that an actual vortex response is being prompted by these temperature changes. In the future, the temperature will need to be recorded as part of possible vortex signals. The LabView code has been modified to record an additional digit in the temperature measurement, so that the effects can be seen in more detail. Controlling them in some way is necessary so that the signals from kelvin waves may be reliably studied without worrying about extraneous temperature signals. This could, however, work to our advantage. With a better understanding of the specific vortex responses that can be caused by temperature changes, the fluctuations could possibly be controlled as another part of the measurement process. This would allow for another tool to study the vortices.

In addition to the interesting temperature-related phenomenon observed, a new question concerning pinning also came up. A vortex that is not bound to its pin-site may have the opportunity to oscillate horizontally along the edge of the cel. A possible test of this phenomenon would be to somehow prompt a vortex that is coming off at the lip of the cell to precess around the lip. If this could be accomplished, we may be able to watch the precession settle down into the original oscillations that we were seeing. This will be difficult however, as pinning on a lip in general is not as well understood as pinning on a bump. If one were interested in pursuing this question further, it would be very useful to create a computer simulation for a vortex pinning to a lip on the cell wall. For the purposes of studying kelvin waves however, using a cell with a more reliable pin-site is probably preferred.

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