Magnetic Phase Transitions in Ising Nanowire Arrays

By Liam McAloon

We use Mean Field Theory, Renormalization Group Theory and Monte Carlo Simulations to explore the magnetic properties of a system of interconnected linear Ising chains. Interestingly in the regular and irregular cases we see non-zero Critical temperatures even for remarkably high dilutions, as well as strong dimensionality.

We began this project as an extension of a project worked on by Edward Burks, a member of Kai Liu's research group. He made a copper foam using metallic track etching followed by chemical etching, that left him with a collection of "nanowires" in an array, that while still solid was made of about 98% air. Figure 1 shows pictures of his creations. Our job was to attempt to make a computer model to figure out what the magnetic properties of a system like this would be like, with our primary focus being on the question of would it demonstrate 2-dimensional or 3-dimensional characteristics.

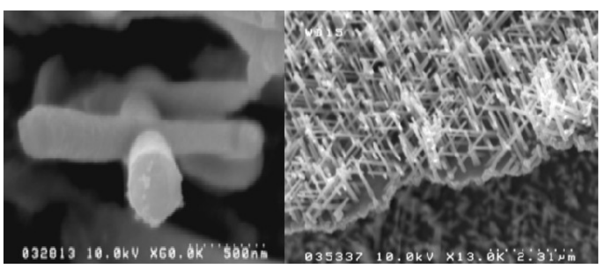

Fig 1: Pictures of the metallic foam. E. Burks and K. Liu, private communication.

The method we decided to use to model this system was to work from a basic Ising model and explore from there. This also gives us a number of tools for determining the dimensionality of the system as we change geometries, as there are very clearly defined temperatures for the magnetic phase transitions in the Ising model in 2-d and 3-d. By comparing our results to the classical numbers for 2 and 3 dimensions we would be able to determine the dimensionality of the system.

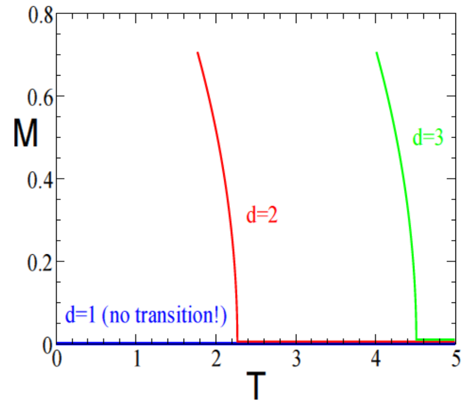

Fig 2: Using a standard Ising model and plotting the Magnetization Vs Temperature gives us the Critical Temperatures for 2-d and 3-d. In 1-d there is no phase transition. The critical temperatures for 2-d and 3-d are 2.269 J/k_b and 4.51 J/k_b.

This sets up most of our basic questions. Does this system have a magnetic phase transition? What are the critical temperatures and what shape does M(t) have? How do they change depending on the dilution of the system? The most critical question being of course what dimension is the system.

To set up the system we started by making a regular 1-d, 2-d and 3-d Ising model with periodic boundary conditions[1], to double check our results against. Then we started exploring a simpler version of the system. Instead of randomly placing 1-d Ising chains like pick-up-sticks, we made a regular grid set up and explored the phase transitions of these at various gap sizes.

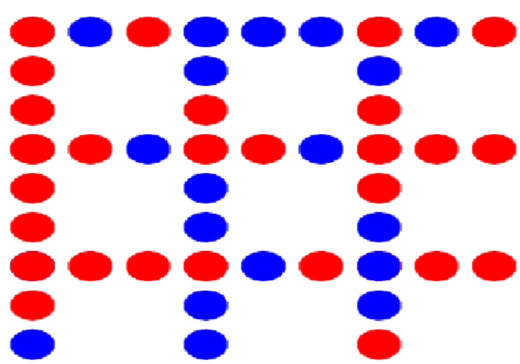

Fig 3: A regular site dilute Ising model with a gap size of 2. Blue is spin down, Red is spin up.

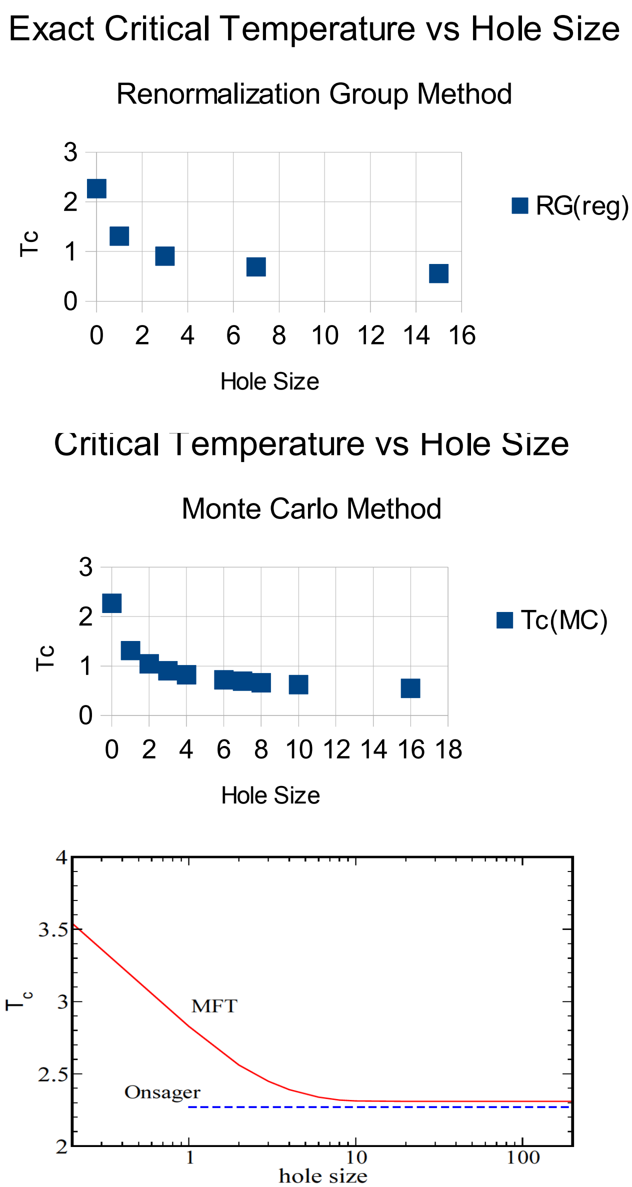

Fig 4: The top graph shows the exact solutions via Renormalization Group Theory.

The next graph shows us the Monte Carlo computer solutions.

The final graph shows the Mean Field Theory approximation and the Onsager solution for 2 dimensions.

We used a number of different methods to determine the critical temperatures. Mean Field Theory provided us with an upper bound for our approximations, and using Renormalization Group Theory we were able to directly determine the Critical Temperature for certain lattice sizes (gap size=(2^n)-1).

To determine the critical temperature with the Monte Carlo simulations we used the Binder Ratio[1], which is a means of finding critical points using the order parameter of the system, which in our case is magnetization. By comparing the plots of the binder ratios at different lattice sizes we are able to very accurately pinpoint the critical temperature.

For ease of use with the Monte Carlo simulations, we defined temperature in our system to be in units of J/k_B. J is the bond strength between nodes and k_B is Boltzmann's constant. This allows for simplifications within our codes and graphs.

The other method to determine the critical temperature is through the Magnetic Susceptibility. When you plot the susceptibility against temperature, there is a peak at the critical temperature. The larger the lattice size you use, the clearer this peak becomes. However, this requires very large lattice sizes and is not overly precise.

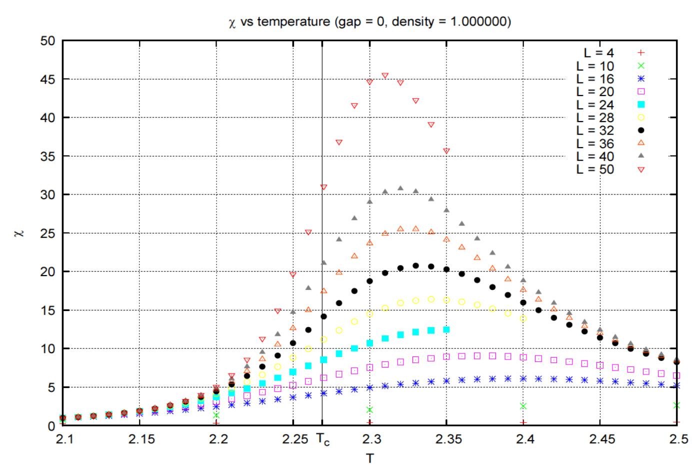

Fig 5: As the lattice size increases, so does the size and definition of the peak.

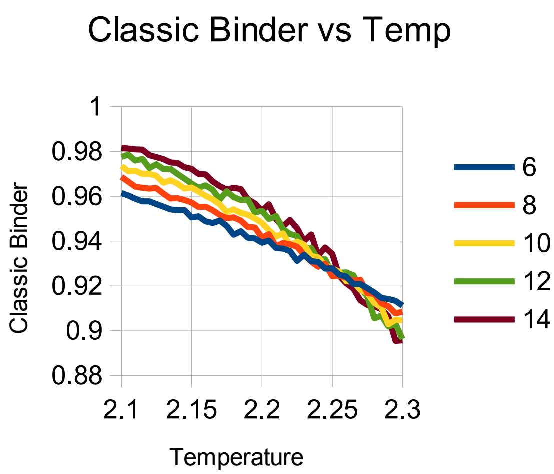

Fig 6: Using the Binder Ratio we are able to see a very precisely where the crossing is, which is the critical temperature. The different colours represent the 1 dimensional size of the lattice.

The next step was to begin working with a randomly generated system where the lines are no longer in a perfect grid. We struggled for awhile to come up with a suitable method for doing this. The final solution that I used was to force the lines onto a grid. By picking a start wall and position as well as an end wall and position I was able to fill in a line on the grid, and by placing these randomly until a specific density was reached I was able to generate random lattices for later use.

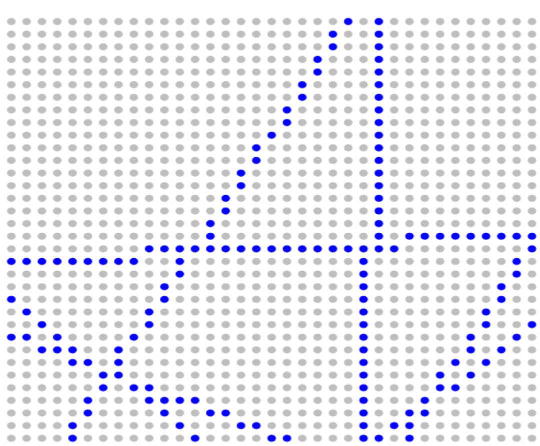

Fig 7: A sample random lattice. Only the darker sites are occupied, the rest are simply there for visual aid.

As we started working with the random lattices problems started to develop. The methods we used to set up the lattices are only effective if the sides of the lattice are greater than 10 and the density is less than ½. However, as the density decreases and lattice size increases this method becomes more and more precise. The other issue we ran into is that the Binder Crossings were not clear and contained so much noise we could not tell what was happening.

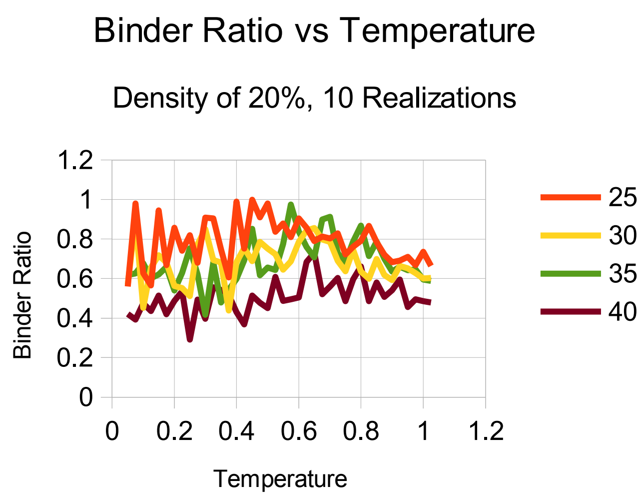

Fig 8: Even using much larger lattices, in comparison to fig 6, and averaging across multiple realizations we were unable to clear up the data.

However, when we looked at the data for the Magnetic Susceptibility we found that we were starting to see a peak in the graph. Using this we were able to start getting data on the random 2-d model.

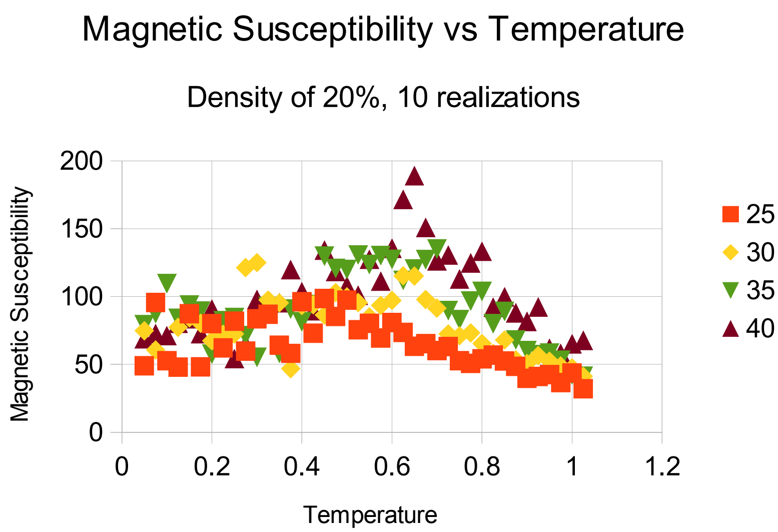

Fig 9: A peak has begun to appear.

Moving forward, we should be able to get more accurate data for 2 dimensions, varying density and increasing lattice sizes. From there we will move into three dimensions, with the long term goal of comparing our work to Edward Burks.

[1]. Katzgraber, H. G., (2011). Introduction to Monte Carlo Methods. *arXiv.* http://arxiv.org/pdf/0905.1629.pdf