Rate and State-Dependent Friction in Earthquake Simulation

Zac Meadows
UC Davis - Department of Physics
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Abstract

To better understand the spatial and temporal complexity of large scale fault zones, researchers are beginning to use earthquake simulators. One such tool is Virtual California, which has been used to generate synthetic seismicity catalogs for various fault zones in a variety of studies. To further improve upon Virtual California, we present the next generation earthquake simulation code called ViCaRS (Virtual California Rate-State). In this work we provide an overview of the ViCaRS simulation code and present some of the algorithms used to improve resolution, performance and accuracy. These include a rate-state model of friction which allows for more accurate rupture propagation and aftershocks, a Barnes-Hut algorithm to approximate far field effects and increase simulation resolution, and a multi-stage solver which provides fast solution of long term fault movement while allowing for fine time resolution of rupture dynamics and tight controls on solution accuracy. We present initial results produced by this code and discuss future directions for improvement.

1 Introduction

Earthquake simulation involves uncertainties at nearly every conceptual level, from microscopic to tectonic. The onset of an earthquake is typically brought about by a combination of many factors, including the constitutive properties of the fault itself, bulk properties of the fault rock, stressing history, pore fluid interactions, and fault geometry. In order for any simulation of such an event to be computationally tractable, simplifications must be carefully decided upon in order to increase speed while retaining accuracy. In the case of Virtual California [1], one of these simplifications has been the friction model, which currently does not extend beyond the principles of high school physics. More realistic and computationally intensive friction models exist. One of them, Rate and State-Dependent friction, will be explored here in the context of Virtual California.

2 Virtual California Overview

Virtual California (VC) consists of three fundamental components: a fault model, a set of Green's functions governing quasi-static interaction of fault elements, and an event model.

2.1 Fault Model

The fault model divides the known faults in California into a square mesh consisting of 3km x 3km elements. Each element possesses a unique position and orientation, a constant back-slip velocity along a fixed rake vector due to tectonic plate motion, and a failure stress. These values are obtained from the Uniform California Earthquake Rupture Forecast (UCERF2) model [2]. In contrast to an actual geologic system where the fault geometry is dynamic, VC employs a static fault geometry. This decision avoids the enormous complexity that arises when modeling element-element interactions within dynamic fault systems, and restricts focus to understanding seismicity in fault systems as they exist now. It is important to note that the fault model is the only component of VC specific to California. The internals of VC have been used with alternative fault models to study earthquake behavior in other geographical regions such as Japan and Thailand.

2.2 Element Interaction

A technique called back-slip is used to model the accumulation and release of stress along the fault plane. Elements are treated like leaf-springs which are displaced away from equilibrium by long range tectonic plate motion. This displacement stresses the element gradually until the element reaches failure stress and instantaneous back-slip occurs. After undergoing back-slip an element is returned to its equilibrium position and the stress released is distributed to other elements.

The degree to which the back-slip of one element affects stress on another element depends on the position and orientation of both elements. These interactions are managed by a set of stress Greens functions, the values of which are calculated using an implementation of Okada's half-space deformations [3]. Due to the static fault geometry of VC, Green's functions values remain constant throughout the simulation and need only be calculated once. Final stresses are calculated by:

$$\sigma_s^A(t) = \sum_B T_s^{AB} \delta_B(t) \tag{1}$$

$$\sigma_n^A(t) = \sum_B T_n^{AB} \delta_B(t) \tag{2}$$

where σ_s^A and σ_n^A are the shear and normal stress on element A, and δ_B is the slip of element B at time t.

T_s and T_n are matrices which store the element-element interaction strengths calculated by the Greens/Okada functions. T_s^{AB} represents how much backslip on element B affects the shear stress on element A. Similarly, T_n^{AB} controls normal stress interactions. Thus, two N x N matrices are required to govern all interactions for a VC simulation containing N elements.

2.3 Rupture Event Model

VC currently uses a simple static-dynamic friction law to determine when element failure occurs. This law is implemented by a Coulomb failure function:

$$CFF^A(t) = \sigma_s^A(t) - \mu_s^A \sigma_n^A(t) \tag{3}$$

where μ_s^A is the coefficient of static friction for element A. When $CFF^A > 0$, element A fails. At this point, stress due to the failure of element A is added to all other blocks in the system with magnitudes calculated using the values of the stress matrices mentioned above.

2.4 Execution Model

A VC simulation begins by calculating the Greens function matrices (Fig. 1). After this, long term tectonic stress accumulation is processed and the time of the first element failure is found. The failed element is then allowed to fail, and its stress is released into the other elements. At this point, if any other element failures are detected, the process is repeated. Otherwise, the simulation reverts back to long-term stress accumulation and the process is repeated.

3 Rate and State-Dependent Friction

The Static/Dynamic friction law employed by VC only crudely describes observed real-world friction behavior. Recently the importance of material history on friction-dependent events such as fractures, slips, and deformations has become of interest in the field of earthquake simulation [4] [5]. Earthquake simulation provides a ready context in which these new ideas may be explored. In general, the current method of reducing the computational complexity of earthquake simulations is to employ an instantaneous onset of slip at some stress threshold followed by an instantaneous recovery of strength after the stress has been dispersed. This is the technique described above and currently employed by Virtual California. Contrastingly, an a object described by Rate and State laws demonstrates time-dependent onset of unstable slip and recovery.

Rate refers to the dependence of the force law for an element on the instantaneous rate of deformation (slip velocity) and *state* represents the dependence of the force law on the history of the material. One may interpret state, in the context of a fault system, as the average age of the load-supporting contacts between two sliding surfaces. Until slip occurs and the contacts are destroyed and recreated, they strengthen with age. This cyclic behavior of stress buildup and release is contained within the rate and state equations:

$$\begin{aligned} \sigma_s &= \sigma_n[\mu_0 + a\ln(V/V_0) + b\ln(V_0\theta/D_c)] & (4) \\ \frac{d\theta}{dt} &= 1 - \frac{\theta V}{D_c} & (5) \end{aligned}$$

where:

σ_s - shear stress
σ_n - normal stress
μ_0 - initial coefficient of friction
a, b - experimentally determined constants
v - displacement rate
θ - *state* variable
D_c - critical slip distance
V_0 - initial displacement rate

3.1 Slider Block Model

In order to implement the rate and state friction model, an equation of motion for the system is required. In the case of an earthquake simulation, one method is to model all elements as spring-connected slider blocks [6]. In addition to their connections to each other, each block is connected to a universal pulling spring that represents long-range tectonic plate motion. Successful simulations for small systems of slider blocks were achieved. For a single block (Fig. 2) the equation of motion is:

$$\left(\frac{\tau}{2\pi}\right)^2 \frac{dv}{dt} = v_p t - x - \frac{mg}{kA}\mu \qquad (6)$$

where:

v - block velocity
v_p - pulling velocity
k - spring constant
m - block mass
A - contact area
μ - coefficient of sliding friction
$\tau = 2\pi\,(m/k)^{\frac{1}{2}}$

4 Results/Further Work

As of now, only artificial and geometrically simple fault models have been simulated using rate and state friction. Preliminary results indicate the slider blocks behave as intended (Fig. 3) in these simple models, following a cycle of stress accumulation and release. Due to the fine-grained nature of block interaction in this model, adequate performance has not yet been achieved and a rate-state simulation using the full Californian fault model is not feasible. A number of methods for improving the speed of a rate-state simulation are discussed below.

4.1 Barnes-Hut Simulation

The Barnes-Hut algorithm is a method for improving the order of a naive n-body simulation from $O(n^2)$ to $O(n \log n)$. The simulation space is recursively subdivided into cubic cells of an octree until each cell contains only one element. In order to calculate the force on a particular body the nodes of the octree are traversed, starting from the root. If the elements contained in an internal node are sufficiently far from the body, they are approximated as a single body.

The resolution of a Barnes-Hut simulation is controlled by an arbitrarily defined parameter, θ. Let s be the width of the region associated with an internal node and d the distance between the body and the node's center of mass. If $\frac{s}{d} < \theta$, the elements within the internal may be approximated as a single element. The position of this new approximated group of elements is their center of mass, and their new mass is simply their aggregate mass. Typically values of θ are less than 0.5, where a higher value of θ decreases the resolution of the simulation and $\theta = 0$ reverts the simulation back to a brute force n-body calculation. The use of a Barnes-Hut style algorithm in a rate-state VC simulation would greatly simplify slider block interaction, in particular for large fault systems.

4.2 Adaptive Time-Stepping

Due to the stiffness of the set of Rate and State differential equations, finding a method of solving that is both quick and stable becomes the primary challenge. One method is to use an adaptive time-step when numerically solving the rate and state equations.

First a threshold velocity for rupture, v_r, is declared. for each time step in the simulation, all slider blocks velocities are checked against this rupture velocity. If any blocks are sliding faster than v_r, the simulation time-step is switched to a much lower value. This allows for more accurate solutions to the rate and state equations during rupture but doesn't force the simulation to adhere to strict accuracy during the long-term non-rupture phase.

5 Conclusion

In order for the slider block model to be used as a viable alternative to the method currently employed by VC, it must become much less computationally intensive. A combination of the simplifications discussed above, along with parallelization, will need to be implemented in order to run the rate-state VC code on the full California fault model. When this is achieved, more accurate predictions will be generated by Virtual California.

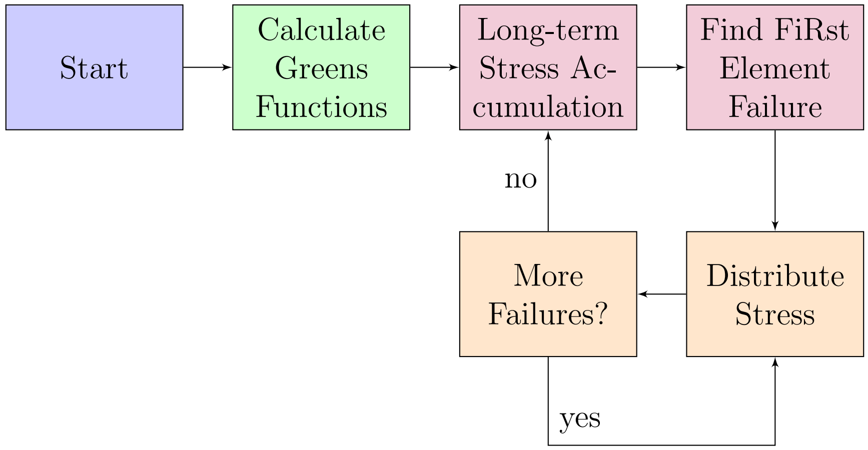

Figure 1: Simplified Execution Flow of Virtual California

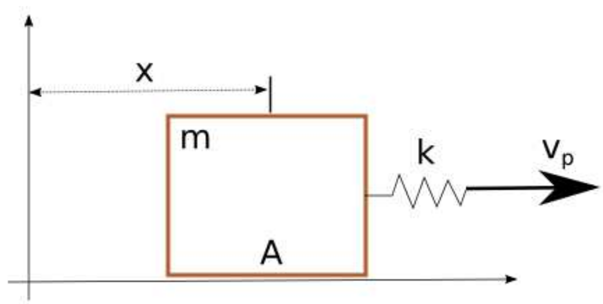

Figure 2: Slider-block model for fault behavior

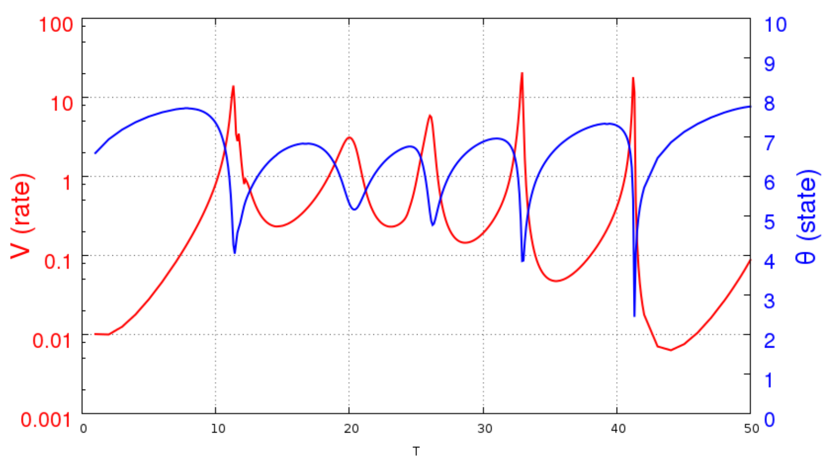

Figure 3: Rate-State evolution of a single slider block within a 20 block system

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