Particle Distribution Analysis of Simulated Granular Avalanches

Abstract

Initial results of two-dimensional, bimodal distribution of grains undergoing avalanches suggests there does not exist a relationship between the angle of stability of a pile of grains and the overall distribution of grains in the pile. We use two types of grain, differing in both size and shape, place them in a rotating drum, and measure the angle of the mixed pile just prior to an avalanche, and attempted to correlate this angle with the distribution of grains. Over 1000 images were analyzed in this manner, and particular care was taken to separate the data of the initial highly segregated mixture from the data of the latter, more random mixture that resulted from the avalanches.

Introduction

Granular materials are large conglomerations of discrete, solid, macroscopic particles. The classical example is sand, but we deal with these materials on an everyday basis; applications range from the food industry (sugar and grains), to construction (concrete mixing), to pharmaceuticals (powder packing and pills), to metallurgy, and many more [1]. Moreover, granular materials may behave, under different densities and motion, as solids, liquids, or gases. For example, a pile of sand on a flat surface, if not disturbed, will behave like a solid. However, if the surface is tilted, at some point the pile will avalanche, flowing like a liquid. Due to properties like these, some suggest that granular materials are a state of matter of their own.

Understanding the relationship between particle distribution and granular behavior is essential in order to comprehend heterogeneous mixtures, which often exhibit segregation between particles of different sizes and shapes. This is especially compelling considering that heterogeneous mixtures appear in most applications (such as the aforementioned concrete mixing or pharmaceutical powder packing).

We set out, in particular, to study the relationship of a binary particle mix distribution and the angle of stability of the mix. The angle of stability of the mix was found to vary between 33° and 54° (*Fig 1*), leading us to suspect that there must be some correlation with the distribution of the mix, especially since the pile can take on very different distributions.

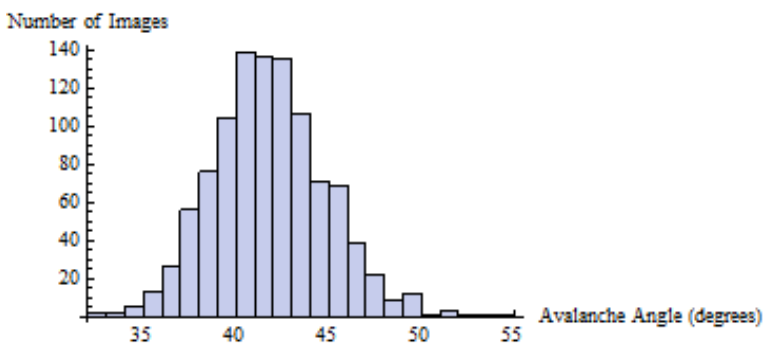

Fig 1: Histogram depicting how many times the pile reached a given angle of stability.

Methods and Procedures

The grains in the mixture were ⅛'' diameter steel ball bearings welded together in twos (dimers) and sevens, creating a hexagonal shape (hexes). The hexes were colored in green nail polish to differentiate them from the dimers; this was important in IDL code that would pinpoint the centers of the hexes.

We put the mixture of 50% hexes and 50% dimers, by weight, in a rotating drum consisting of ⅛'' thick aluminum sheet with a 13.96'' diameter circle cut out of it. Two .5'' thick Plexiglass sheets are screwed on either side of the aluminum sheet. The circular region has a red background while the outside is white; the contrast aids in image processing. It is important to note that, while there may be some friction on the grains due to the pressure of the Plexiglass sheets, it is not believed to be a major factor. However, by confining the grains to a plane, we are able to discern the distribution much more easily, as well as simplifying the problem just by the lack of another degree of freedom.

The mixture is placed into the rotating drum via a slot on the side of the aluminum sheet. The slot is closed while the drum is rotating. Particular care is taken to ensure that the dimers and hexes are not broken upon entering the drum; they are slid down into the drum on top of a wire. Over the course of the experiment, only one hex and two dimers broke, which were promptly replaced. Since all the dimers were placed into the drum and then the hexes, we had introduced an artificial segregation in the pile (*Fig 2a)*. This segregation lasted up to an hour after the drum started rotating, but it took up to two

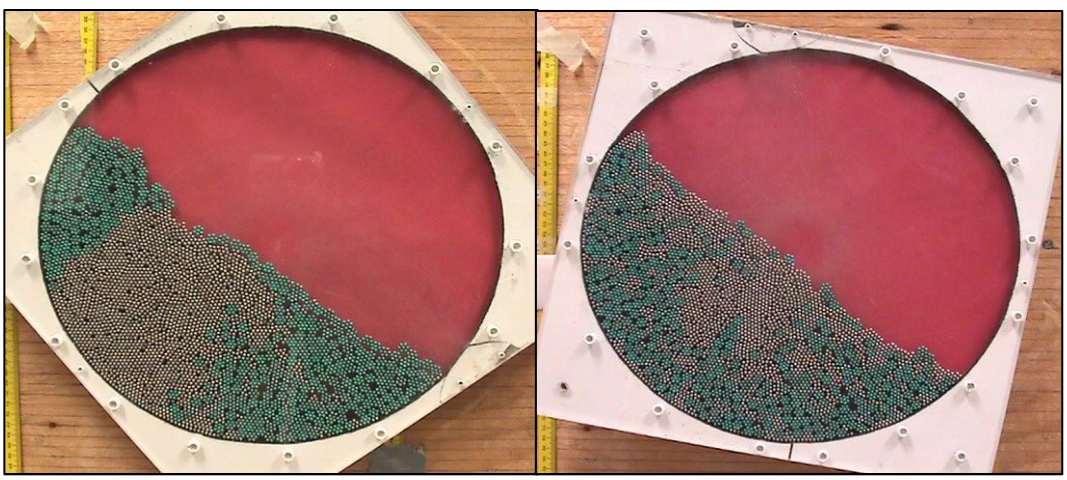

Fig 2a: The "transient stage" is characterized by its clear, artificial segregation.

Fig 2b: A typical hex configuration in during the "steady state."

hours to reach a "steady state," (*Fig 2b)* which was characterized for its much more random distribution of particles. We note that even in the steady-state, the particles exhibited segregation, with the dimers preferring to pool in the center of the pile.

A stepper motor controls the rotation of the drum. The drum rotates at 500µHz, or a full turn in slightly over half an hour. An avalanche occurs when the pile rotates into an unstable configuration, leading to a

flow of particles; on average, an avalanche occurred within 40 to 60 seconds of the previous avalanche. Sometimes, however, only a handful of particles would flow down the pile; these events were excluded from analysis, as the difference in the angle of the pile before and after such avalanches was less than one degree. The data was taken using a Sony DCRTRV30 MiniDV Camcorder, set upon a tripod, which filmed the rotating drum for an hour at a time. A total of eight hours of data was taken, four hours of steady state data and four hours of the early, "transient" data. We upload the 720x480 frames right before and right after the avalanche to a computer, though only the "before" pictures were analyzed in this research. It was found that, when the images were uploaded, they were stretched in the horizontal direction. A Python program was written in order to compensate for this stretch, as well as movements of the tripod in between recordings, which could lead to a displacement of the circular drum or augmentation or reduction of the size of the drum in the image.

Analysis and Results

A previously written IDL program was used to compute the angle of the pile for all the images. In addition, another previously written program, coded in C, was responsible for analyzing the images and pinpointing the centers of the hexes, outputting a list of the coordinates of the hex centers (with the origin placed at the lower left corner of the image) for each image. Then, ordering the images by angle of stability, these lists are concatenated, 30 images in a pack, and another program counts the number of instances that a hex landed on any coordinate. Finally, we output a "heatmap," (*Fig3*) which is an image of the average of where the hexes where found over all the images in the pack. The variable in the heatmap is simply the number of times a hex was found, and the warmer colors correspond to more instances of found hexes.

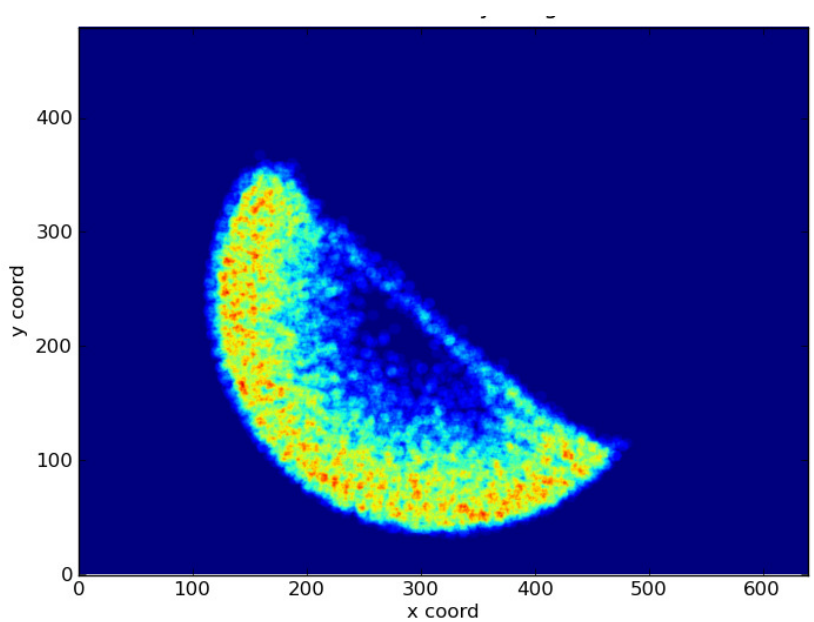

Fig 3: A typical heatmap output. Notice the center is nearly devoid of hexes; this is due to the natural segregation between the hexes and the dimers.

For the “transient” stage, the heatmap method actually destroyed some of the information from the images. For example, a heatmap that exhibited a fairly even distribution of hexes along the edge of the pile could correspond to a bimodal distribution of hexes pooled at the top and the bottom as well. The only way ascertain which distribution corresponded to the heatmap was to look at the original pictures. Keeping this in mind, we found that the data at first indicated a correlation in which hexes pooled on both sides of the drum corresponded to a high angle of stability (*Fig 4a*). However, further data sets refuted this assertion (*Fig 4b*).

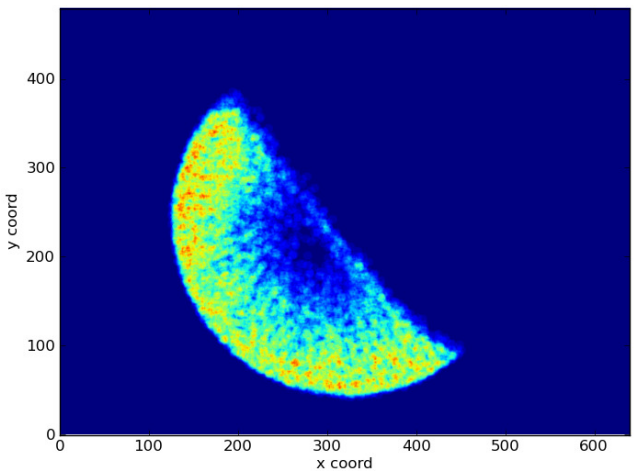

Fig 4a: Heatmap for the first transient data set. Notice hexes pool at both ends of the pile.

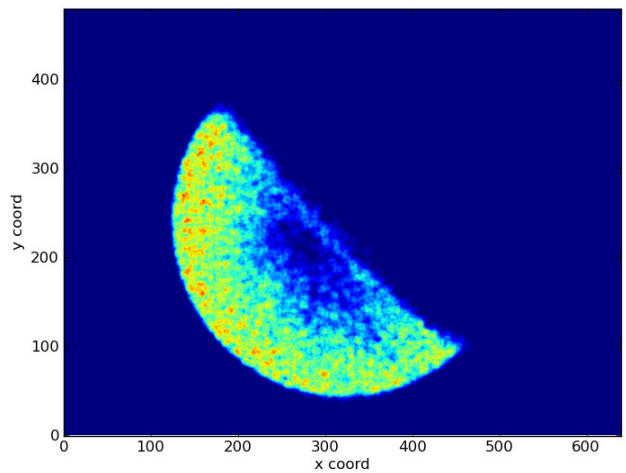

Fig 4b: The heatmap for the second transient data set. Here, the hexes prefer to pool at the upper left end of the pile.

Indeed, it seem that, for the transient stage, there wasn’t a clear correlation between the distribution of hexes and the angle of stability of the pile. For the “steady state” stage, there similarly didn’t seem to be a trend between the distribution of hexes and the angle of stability. However, we noticed two qualities in the images that we hoped would be interesting.

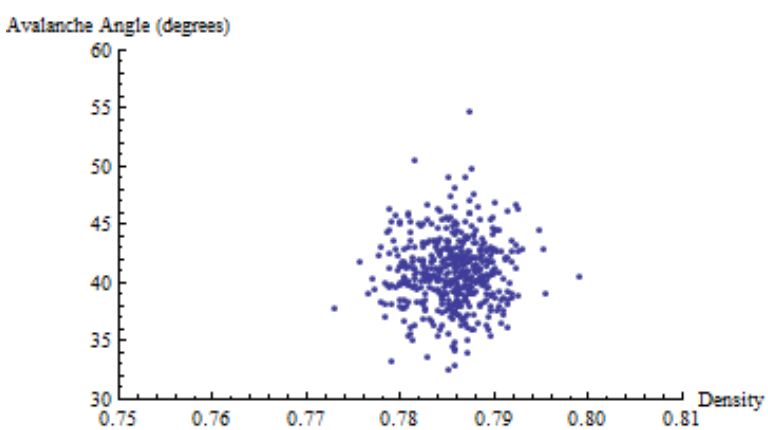

Fig 5: Correlation Coefficient: 0.0214584

Firstly, it seemed that whenever the hexes were densely packed, the pile would reach a high angle of stability. Analysis was carried out, using a program to compute the density of the pile in every image, and plotting the angle of stability versus the density of the pile. The resulting scattershot plot indicated that there was no correlation (*Fig 5*).

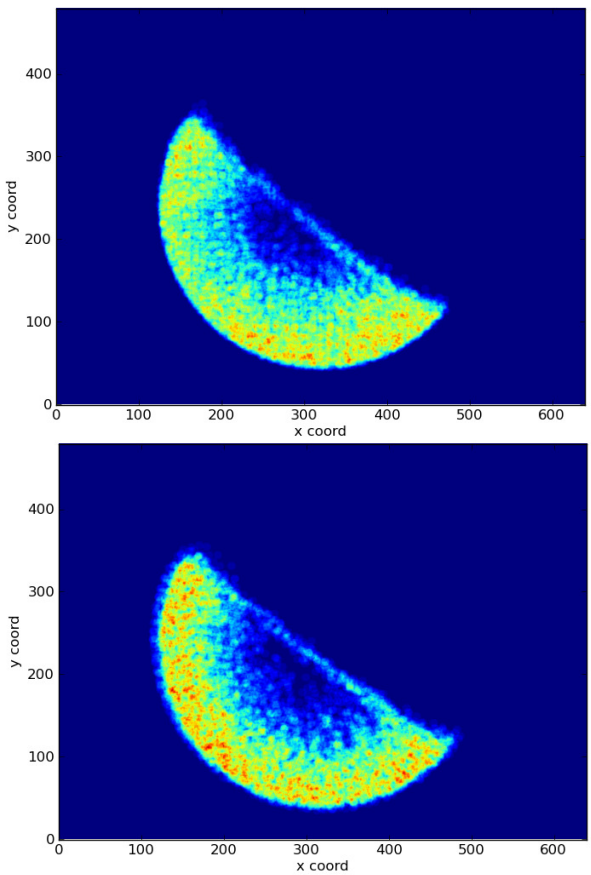

Fig 6a (Top): Transient, Low angle
Fig6b (Bottom): Steady state, Low angle.
Notice both images have a thin line of hexes running along the top of the pile.

Secondly, we noticed that there seemed to be a thin line of hexes along the top of the pile for the low-angle images, in both the "steady state" and the "transient" stages (*Fig 6a,b*).

We wrote an IDL procedure to count the hexes that appeared along the top of the image, using information we already had about the locations of the hexes. Again, we plotted this information versus the angle of stability, but the scattershot plot again denied the existence of a correlation (*Fig 7*).

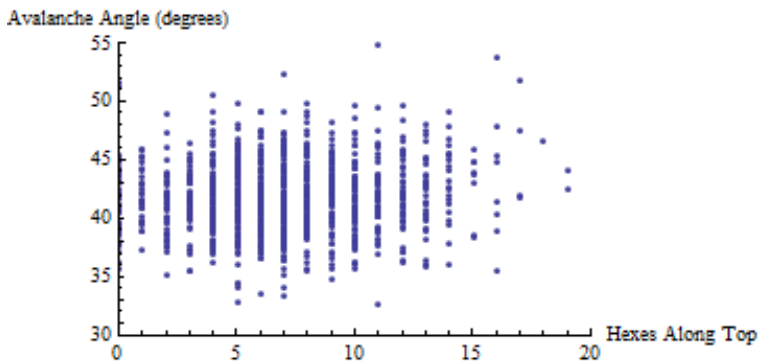

Fig 7: Correlation coefficient: 0.0936586

Conclusions

While we may not have found a correlation between particle distribution and angle of stability, we have ruled out some possibilities. It would be useful to investigate more localized triggers for the avalanches. That is, instead of seeking to correlation the overall density of the pile versus the angle of stability, investigate instead the local density of the pile around the origin of the avalanche versus the angle of stability. Similarly, one could look at the local distribution of particles near the origin of the avalanche. For this, we would require a high-speed camera in order to catch the exact frame right before and right after the avalanches, giving a clearer picture than our current camera can produce.

References

[1] R. M. German, *Particle Packing Characteristics* (Metal Powder Industries Federation, 1989).

[2] A. G. Swartz, J. B. Kalmbach, J. Olson, R. J. Zieve, *Segregation and Stability of a Binary Granular Heap*. Granular Matter, 11(3), (2009).