Binary Mixtures of Two-Dimensional Granular Piles

Kasey Kelly
Dr. Rena Zieve
UC Davis REU Program 2008

October 19, 2008

Abstract

Previous work on the maximum angle of stability of granular material piles had shown a dip pattern, with lower angles of stability for mixtures of hexagons and dimers or singles made out of ball bearings. This purpose of this study was to confirm this pattern for mixtures of different shapes, namely doubles and large diamonds and small and large triangles. This was done by recording footage of the mixtures rotating in a two dimensional drum and analyzed using an IDL program. The results provided conflicting data and it was found that there had been problems with the assumption of planarity for the current data runs. Analysis of how to detect non-planarity were expanded, and a new design of drum was created as an attempt to minimize the presence of a third dimension.

Introduction

Granular materials are a ubiquitous form that matter takes consisting of macroscopic particles combined into one material. These materials may be mixtures of many different shapes and sizes, such as sand or livestock feeds. Depending on the size and shape of the container and the motion used in agitation, a granular material may exhibit the properties of a solid, liquid and gas phase, yet do not undergo a phase change. Through agitation a mixture of different sized grains will exhibit self-segregation. For the same mixtures changing the container size or shape may change the way the particles segregate. [3]

In most areas of modern life there are cases of granular materials: geology in mudflows and avalanche patterns; pharmaceuticals in pill segregation; agriculture with feed pressure and movement in silos, and how produce sorts in transit and packaging; industry in creating briquettes for sintering and brickmaking. Through further understanding the fundamentals of granular materials there are countless benefits across many fields.

In industry, each mixture of materials has been found to exhibit it's own characteristics in vibratory compaction as well as flowing quality [2]. Mixtures that may exhibit the same qualities under vibratory compaction may not exhibit the same qualities under presser compaction. However, most past research on stability and compaction of granular materials has been for the industrial purposes of sintering and powder compaction. From these studies it has been shown that particle size in mixtures holds a noticeable effect on compaction ability, which is also correlated to high repose angles in loose pourings an higher packing densities [1]. The angle of repose is the angle with a horizontal that a granular material pile will make when it has been loosely poured through a funnel. However, granular mounds can get higher than this angle and the maximum angle which a pile can form to the horizontal before collapsing to the repose angle is the angle of stability.

Experiments with vibration and rolling compaction have found that there is a relationship between how stable a configuration is and the packing density [1]. Packing density can be thought of as being related to the amount of pore space the shapes have in a given configuration compared to the perfect for the given shape. When there is minimal space between given shapes the density is greater and the tessellation of crystallization of grains is maximized. In previous works it has been shown that maximally packed briquettes are more stable than less dense briquettes [2]. The way in which most shapes have their maximum packing density is the natural tessellation formation, and when shapes are in these configurations they should be able to better distribute the forces applied to them. Since extra pores would indicate an irregularity in the the lattice structure, it would be expected that the stability of a mound of mixture with a higher packing density would be less than one with a lower packing density.

In past experiments in this lab, it was found that piles of granular materials of only one shape were more stable than mixtures of two shapes. This was done with mixtures of hexagons and singles, and hexagons and doubles. Both studies had yielded a curve that was concave up. Work had been done the previous summer with doubles and large diamonds. The data for the doubles and large diamonds had a noticeable curve, however there were jagged points that needed to be redone to check the validity of the current points. Previous work has also looked into the packing fraction (density related to maximum achievable density) and angles for homogeneous piles. [5][6]

The questions posed for this research were:

1. Why are mixtures inherently less stable than homogeneous shapes?
2. Is the parabolic trend seen in the previous data true for all binary mixtures?
3. What (where and how) initializes an avalanche?
4. In mixtures, what makes the segregation pattern that is seen (smaller particles congregating in the top-center of the mound)?

In order to see the stability of the grains and simplify the process a two-dimensional set up is used. This represents a portion of a pile; only a portion of one side of a naturally forming pile is seen. From this cut away the actual process of an avalanche can be seen.

The mixtures run used different size ratios to observe an effect of size on this form of granular agitation and how it compares to the effect of size on vibratory compaction and rolling compaction of which there is vast data.

Experimental Procedures and Set Up

The set up consists of a rotating drum created by an $\frac{1}{8}^{th}$ inch sheet of aluminium with a circular hole with a 14 inch diameter cut into it. This is sandwiched between two pieces of $\frac{3}{8}^{th}$ inch plexiglass. The inside of the circle is coated with rubber to reduce sliding and promote more realistic avalanche behavior. The aluminium portion is covered in white, and the background of the drum is red to assist with the image recognition software. The drum is rotated by a stepper motor set to 200Hz causing roughly one rotation every 20 minutes. This can be seen in Figure 1.

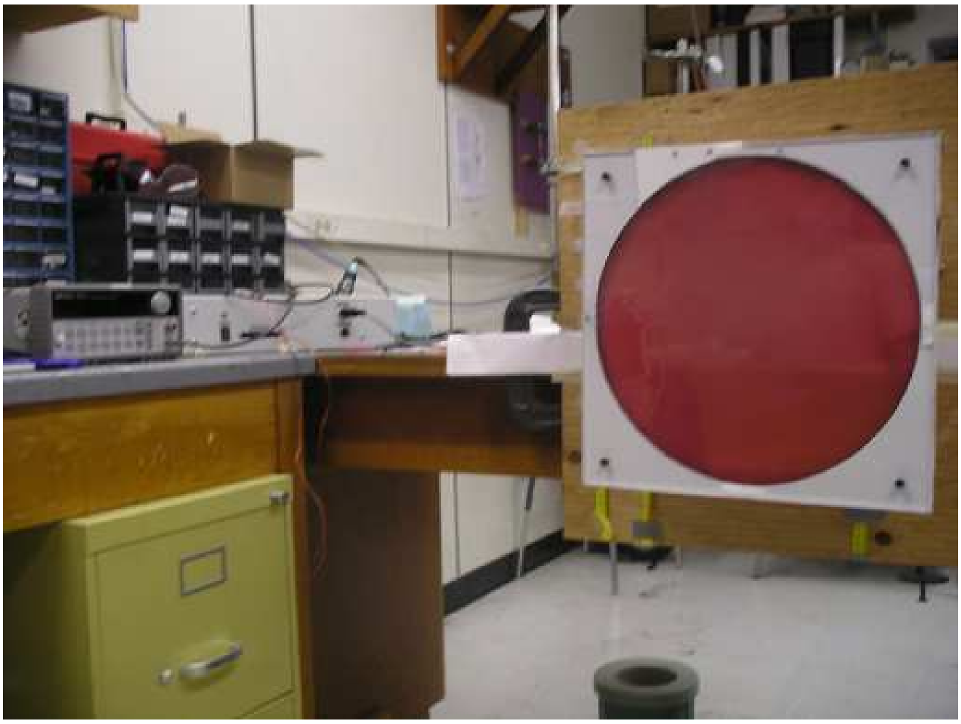

Figure 1 Rotating Drum Set-Up

The grains are created by spot-welding $\frac{1}{8}^{th}$ inch steel ball bearings together into planar shapes. Careful creation of the balls and making sure each ball is welded to at least one neighbor significantly reduces the amount broken in a run. Newly made shapes are run through the drum for an hour to test their stability before being used in normal data collection runs. The shapes used in this study were singles; dimers; small triangles of three balls; large triangles of six balls; hexagons of seven balls; and large diamonds of nine balls, seen in Figure 2.

Each data point required taking 575g (half full drum) or 365g ($\frac{1}{3}$ full drum) of balls and loading them into the drum via a wire to reduce the impact while loading. Mixtures of grains were taken by mass. Using a mini-DV camcorder, the drum is filmed for 2 hours. The first 15 minutes of this is used as "settling" so that the data used is not influenced by how the grains were loaded. If a run finishes with 2% or greater of the shapes by mass broken, the data is not used. Excess and randomly shaped smaller particles would impact the results found for mixtures of large and small shapes. Using *Kino* the still frame immediately preceding and following an avalanche is taken. These images are then run through an *IDL* program that distinguishes the red and white border from that of the grey of the balls. The boarder line is then averaged and that average is taken against a horizontal to determine the critical and repose angles respectively.

Figure 2 Ball Bearing Configurations Used

Because the avalanches are being taken by eye, there is at times discrepancy in distinguishing what is an avalanche from what is “settling”, or a small amount of balls running over the top of the pile. The most prominent difference between settling and avalanching is the fluid behavior of the top layers moving over a stationary bottom portion, where the settling does not exhibit this fluid behavior. Previous work found a suitable cut-off for avalanches is a difference of 4° between the critical and repose angles. This cut-off was used and averages did not include avalanches that had differences of less than 4°.

An additional set of IDL programs finds the bright spot on each ball and from this calculate the distance between any ball and all other balls. The distance in pixels between the centers of each pair of balls is taken as a bin number forming a histogram. The programs also calculate the ratio of ball-radius-to-pixel and from this a histogram of the ball-diameters away all of the balls are can be found.

Results

The ratios of small:large of the particle mixtures used and referred to.

Small Shape	Large Shape	Ratio
Single	Hexagon	0.143
Double	Hexagon	0.286
Double	Large Diamond	0.222
Small Triangle	Large Triangle	0.5

Table 1 Size Ratios of Shapes Used.

Doubles and Large Diamonds

The initial data for the doubles and diamonds mixtures where all data taken for these runs used 575g of ball bearings. The curve in Figure 3 shows the rough concave up trend in the stability angles. The repose angles are shown in Figure 4, and the packing fraction for the initial stability angles is shown in Figure 5.

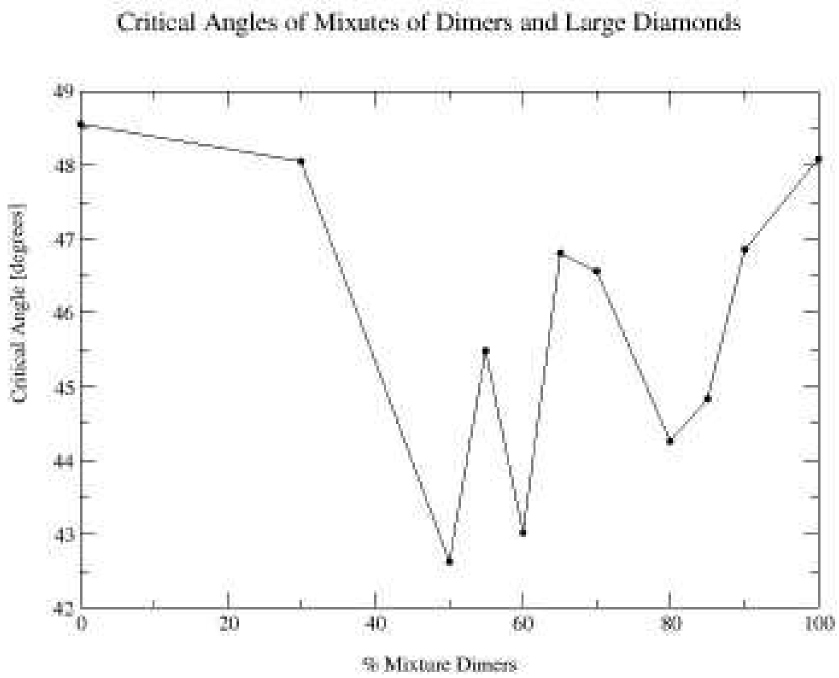

Figure 3 Initial Average Stability Angles for Doubles and Large Diamond Mixtures

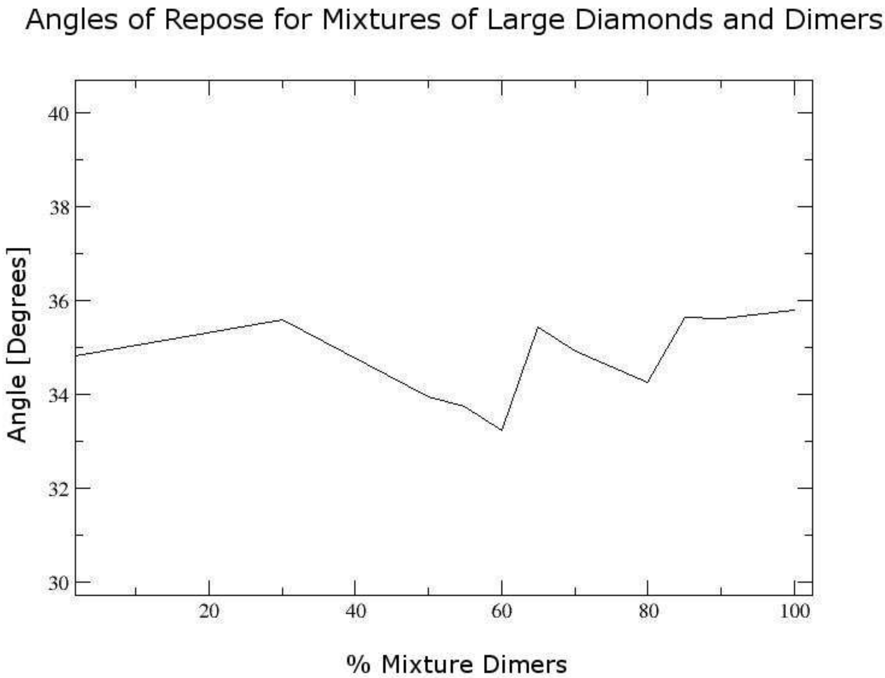

Figure 4 Initial Average Repose Angles for Doubles and Large Diamond Mixtures

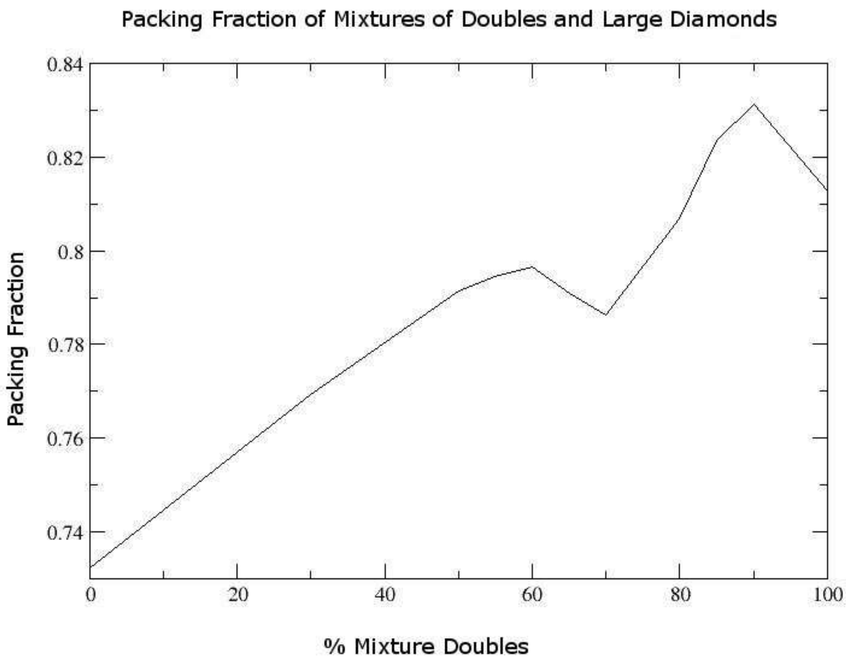

Figure 5 Initial Densities for Critical Angles for Doubles and Large Diamond Mixtures

Data taken after rerunning data points for stablility angles in Figure 6, repose angles Figure 7, and packing fraction in Figure 8.

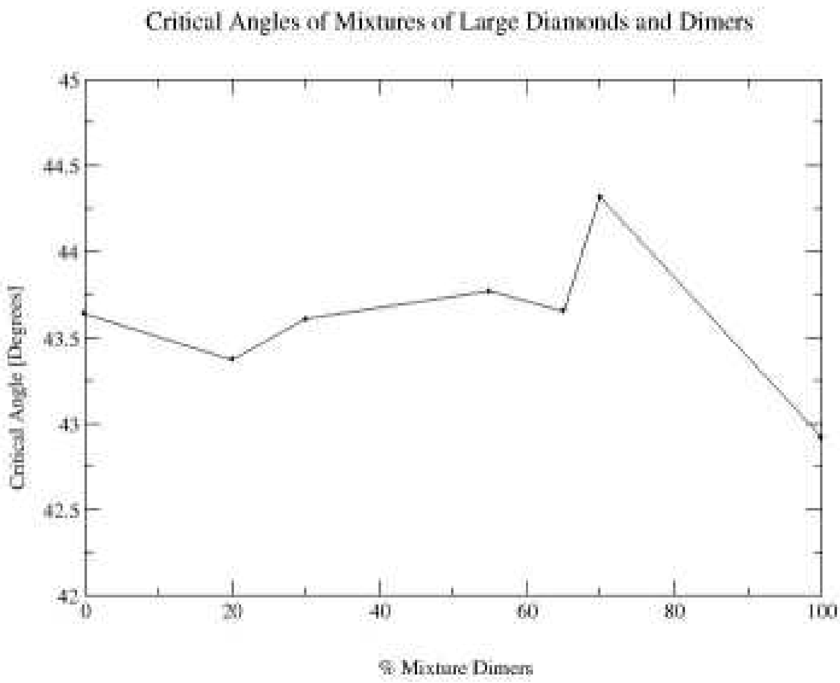

Figure 6 Redone Average Stability Angles for Doubles and Large Diamond Mixtures.

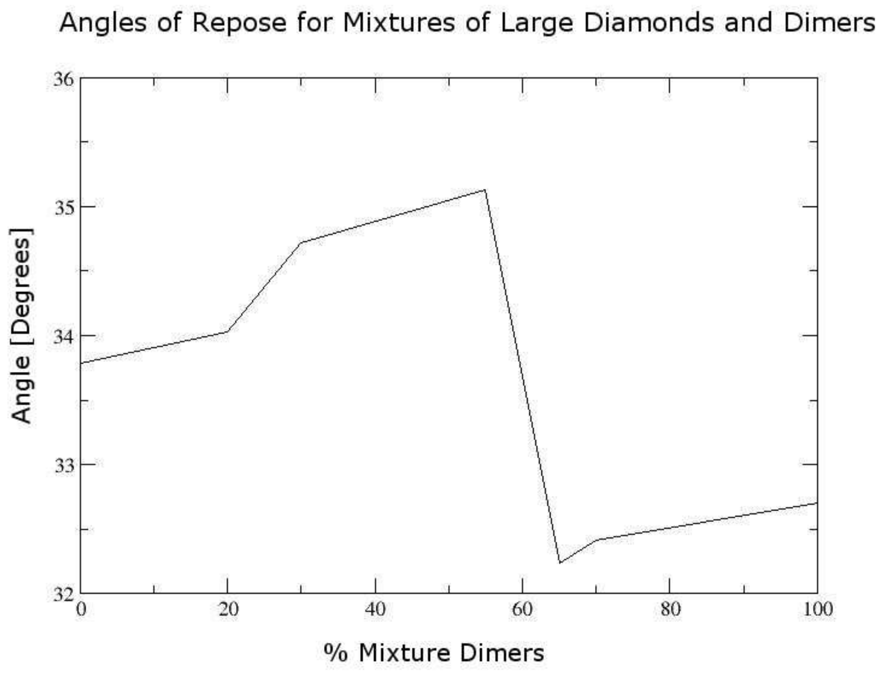

Figure 7 Redone Average Repose Angles for Doubles and Large Diamond Mixtures.

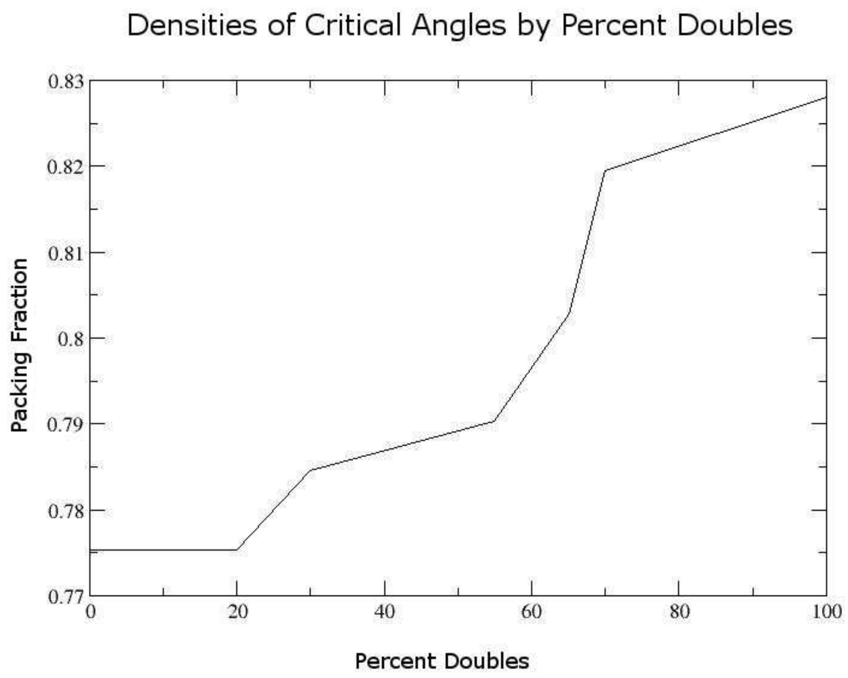

Figure 8 Redone Densities for Critical Angles for Doubles and Large Diamond Mixtures

The angles of stability, repose and packing densities for the end points of Figures 3 and 6 used to gauge differences through time in Table 2.

% Doubles	Critical Angle	Density	Repose Angle	Density
0 - Before	48.6	0.732	34.8	0.733
0 After	43.6	0.775	33.8	0.758
100 Before	48.1	0.81	35.8	0.807
100 - After	42.9	0.828	32.7	0.834

Table 2 Redone Average Angles for Doubles and Large Diamond Mixtures.

Comparison of the individual angles spread for the initial all double runs done immediately before and after the initial tank adjustment, checking for spread of angles and occurrence of outliers. Shown in Figure 9. The two lines had average angles of 48.8° for the initial run (blue) and 45.3° for the after run (green).

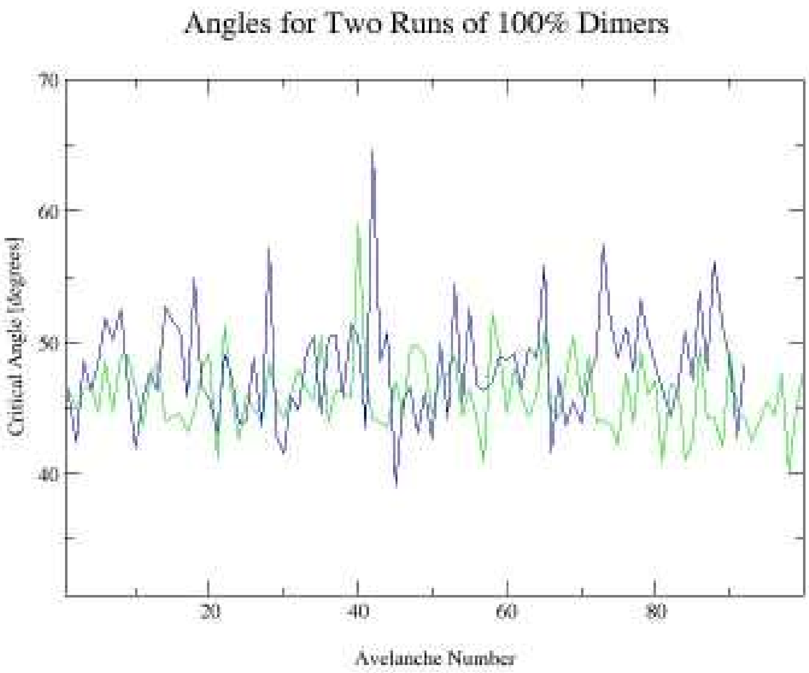

Figure 9 Spreads of Doubles Runs before tank adjustment (blue) and after (green).

Triangles

Initial data for the small and large triangle mixtures was taken in an alternating order of, by % small: 100, 0, 50, 85, 15, 65, 35. All data taken was with 575g of ball bearings. Figure 8 shows the stability angles, Figure 9 the repose angles, and Figure 10 the densities for the stability angles for the initial data run.

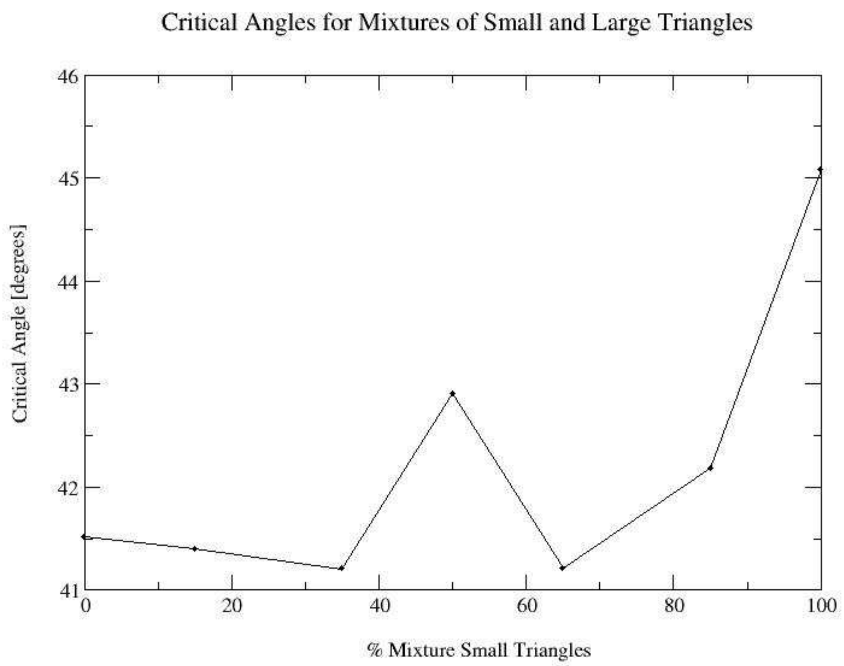

Figure 10 Initial Average Stability Angles for Small and Large Triangle Mixtures

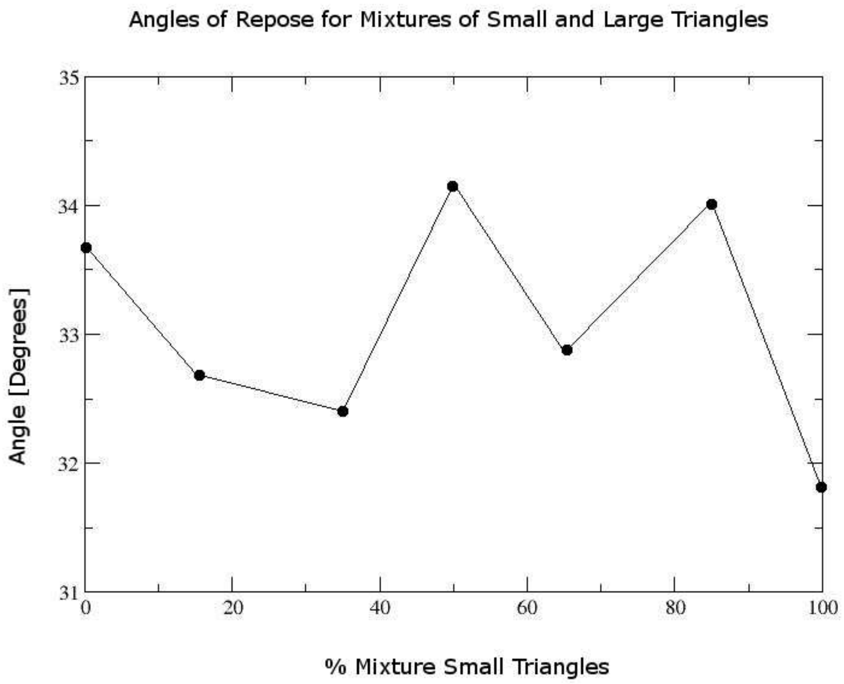

Figure 11 Initial Average Repose Angles for Small and Large Triangle Mixtures

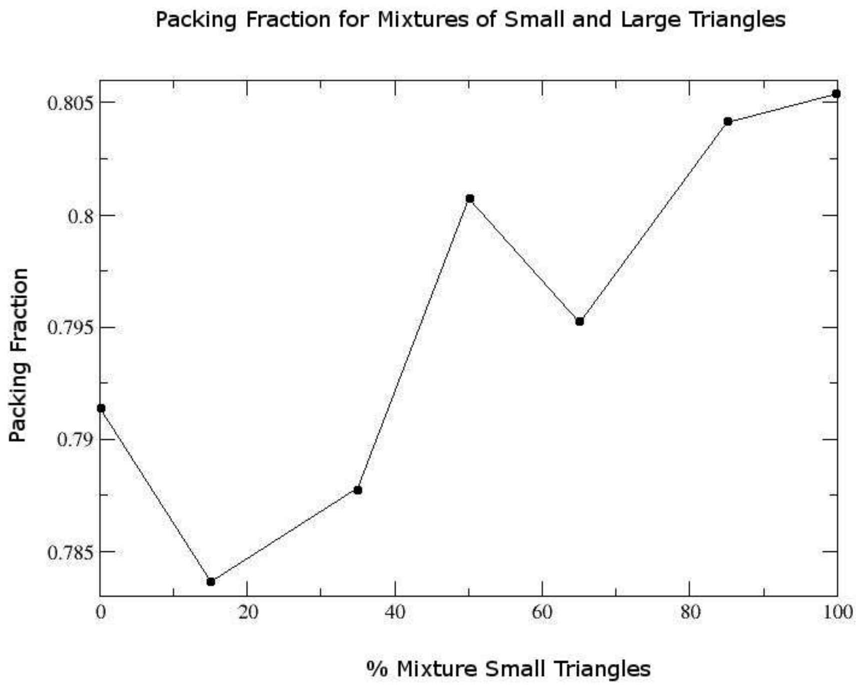

Figure 12 Initial Densities for Stability Angles for Small and Large Triangle Mixtures

Redone data points were taken in the order (by %small triangles): 0, 100, 50. The initial data is indicated with blue dots, the additional data points with green dots, and a prior data point taken before the first drum realignment with purple. Figure 11 shows the angles of stability, 12 angles of repose, and 13 the packing fractions.

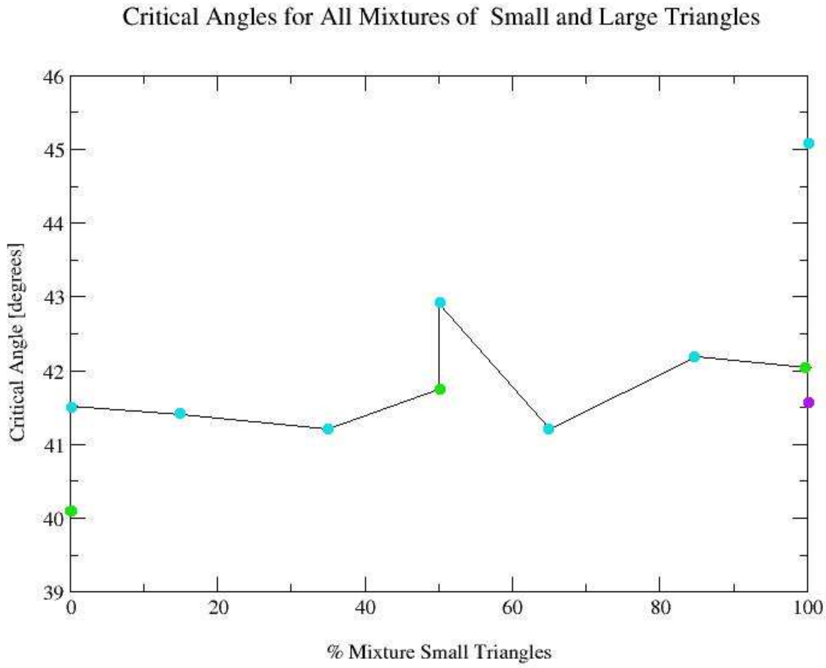

Figure 13 Initial Average Stability Angles for All Runs of Small and Large Triangle Mixtures

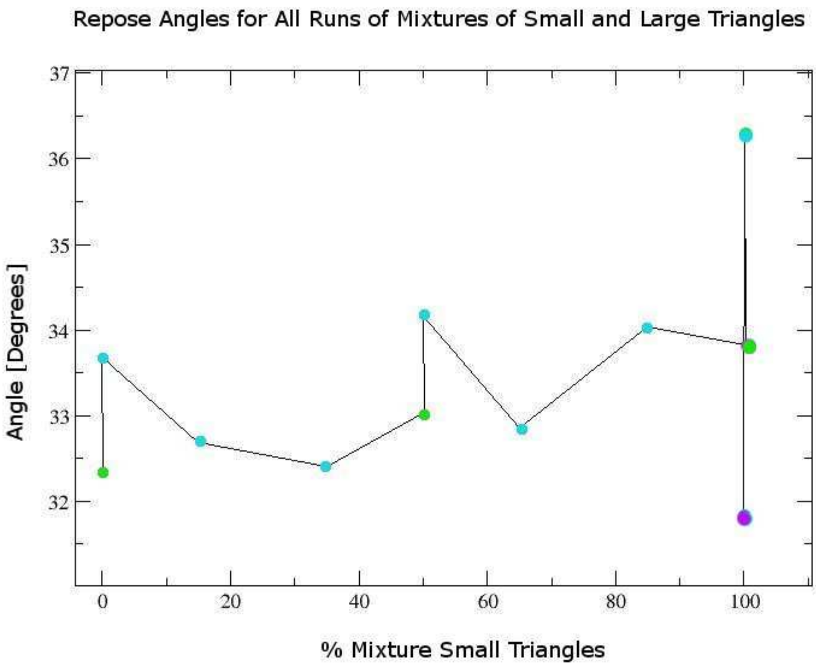

Figure 14 Initial Average Repose Angles for All Runs of Small and Large Triangle Mixtures

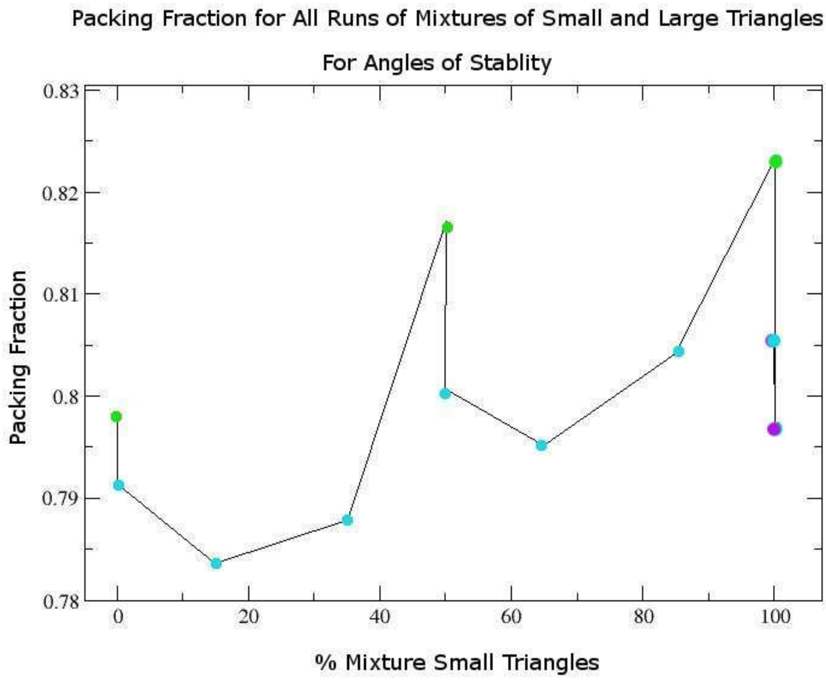

Figure 15 Initial Densities for Stability Angles for All Runs of Small and Large Triangle Mixtures

Hexagons and Singles

This data was initially taken in previous experiments and subsequent data was taken and compared. All data was taken with 365g of ball bearings.

Comparison of the individual angles spread for the initial all double runs done several years before and after the initial tank adjustment, checking for spread of angles and occurrence of outliers. In Figure 16 the spread for data taken for mixtures of 10% singles and 90% hexagons is shown. The new data (black) had an average angle of 47.0° and the old data (red) had an average angle of 43.0°.

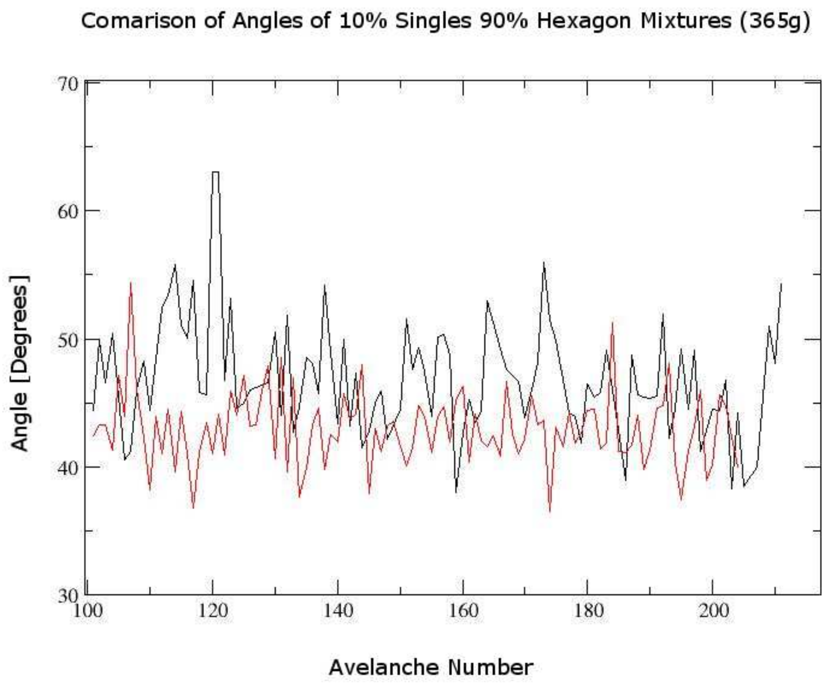

Figure 16 Angle spread comparison, black line indicating new data, red line indicating old data.

In Figure 17 the average angle for the new data (red) 53.3° and the old data (black) had an average angle of 47.7°.

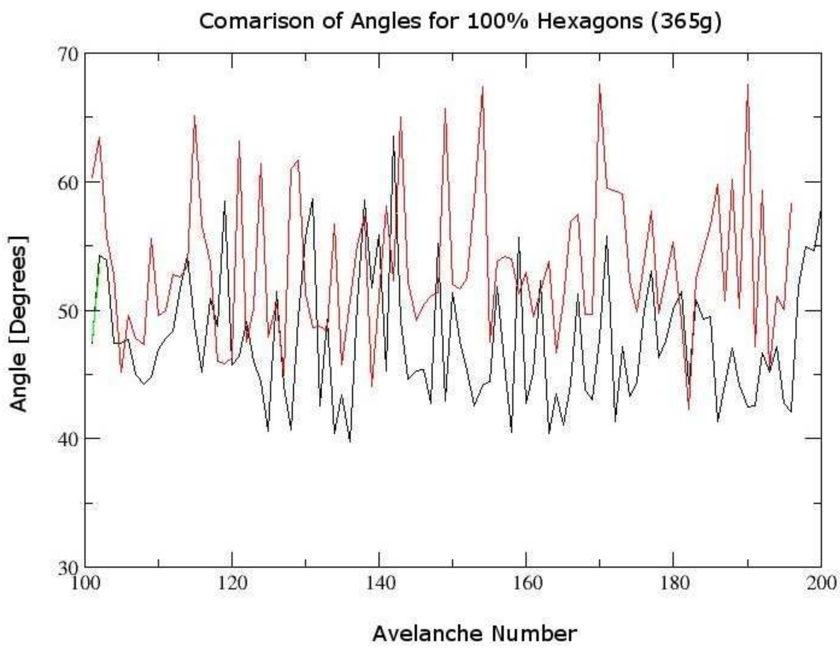

Figure 17 Angle spread comparison, black line indicates old data, red line new data.

% Hexagons		Critical Angle
90	Old	43.0
	New	47.0
100	Old	47.7
	New	53.3

Table 3 Average Angles for Singles and Hexagons Mixtures.

Histograms

Histograms were made of the distance from every ball center to every other ball center. The occurrence of each distance was recorded. The range of the first two ball diameters was studied.

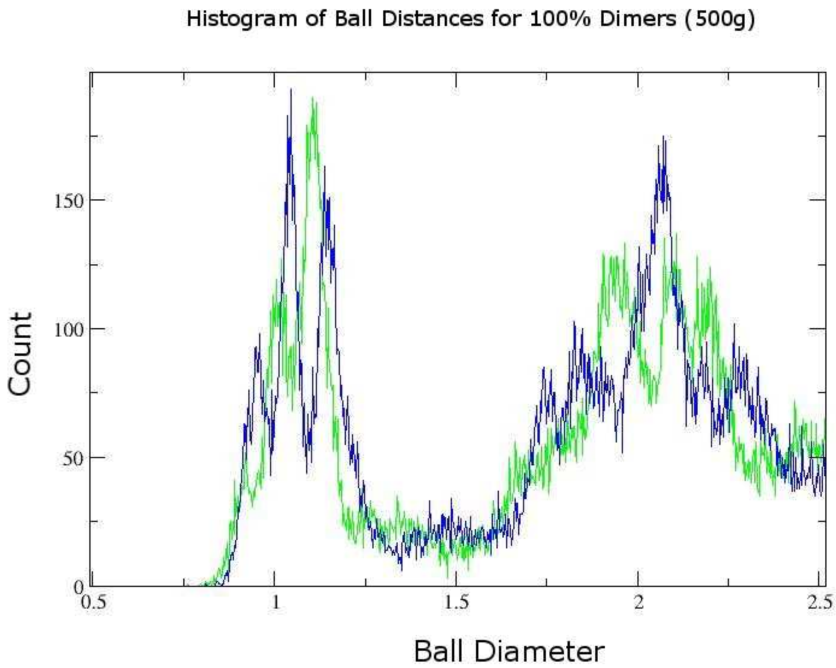

Figure 18 Histogram of ball diameters for runs of 100% doubles; green line a run before drum realignment, and the blue line after.

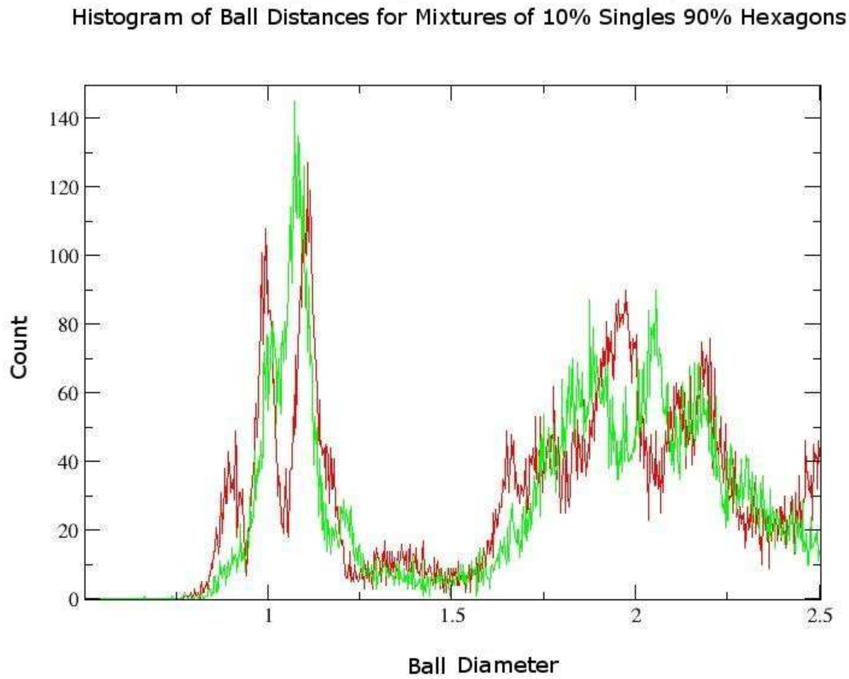

Figure 19 Histogram of ball diameters for 90% hexagons runs; green line indicating data taken perviously, and red new data.

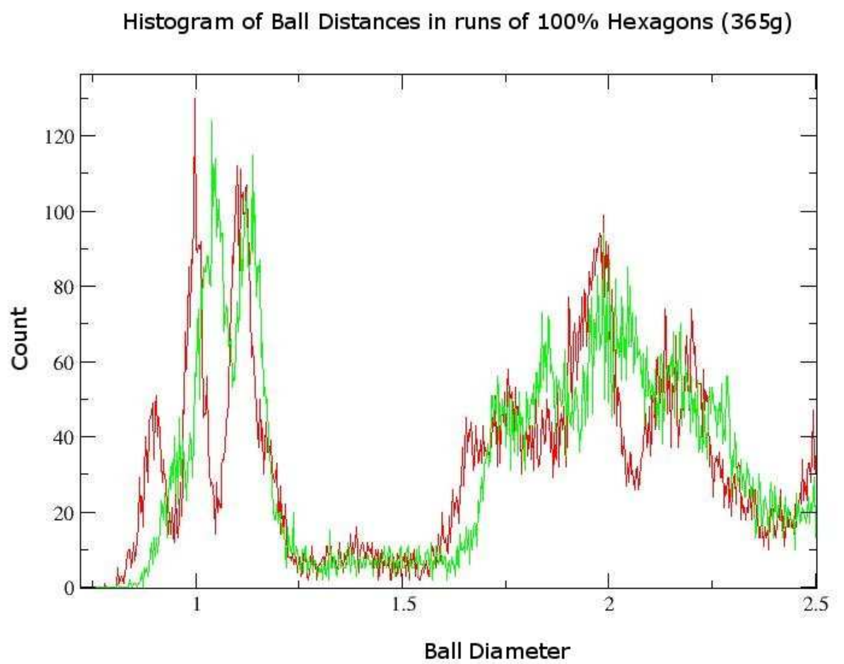

Figure 20 Histogram of ball diameters for 100% hexagons runs; green line indicating data taken perviously, and red new data.

Analysis & Discussion

During the data taking, there were many impedances to data aquisition, most noticeably image quality. From the initial data taken to data taken several weeks later the image quality from the camera deteriorated. When checked against data taken one year or more before, the image quality was more grainy and pixilated, which interfered with the program's ability to find the center of the balls, and also seemed to be affecting where the line used for determining the slope was appearing at times. Although it was not apparent which method restored the image quality for playback, running new tapes at fastforward and fast-rewind as well as cleaning the tape heads were tried. It was found that all data had been recorded at high quality so several key runs may be re-run to determine if problems found were exacerbated by the low image quality.

The camera contrast also affected the way the program handled the data. Several tapes made prior to this study that were used had different contrast than previously run data. This was attributed to not having all lights on in the room, only the set directly over the drum. A change was made to the IDL code so that the contrast of the image that the program did analysis on was adjusted accordingly. This was done by putting a cap on the upper brightness of any colour array value at maximum to prevent wrapping. This allowed many more data runs to be analyzed in a more uniform manner. It also enhanced the programs ability to detect the center of the balls and the white edge, reducing errors.

The initial doubles and large diamonds data had the trend of a dip in the angle of stability as the mixture content reached half and half, Figure 3. However, there were several outliers to the trend. The points causing the outliers had been redone and when they were analyzed they did not match the predicted pattern. Redoing several other data points yielded the relatively flat line in Figure 5. The data has a spread of less than $2°$, which indicates little variation in any of the mixtures. The difference in intimal data and verification data was drastic, yet apparently systematic: the verification data was $5°$ less than the initial data as shown in Table 2.

The systematic angles led to analysis of the densities for the respective sets of critical angles, Figures 5 and 8. From this it was concluded that there were problems with the assumption of planarity of the drum. It follows that if a non-trivial amount of third-dimension is added the particles will pack to be more dense than if they were in a purely two-dimensional plane. Figures 5 and 8 show that there is an addition of non-trivial third-dimension: Figure 8 shows a net higher packing fraction (better packing is represented by values closer to 1) than Figure 5, showing that there is more of a presences of a third dimension in Figure 8 than Figure 5. As seen in previous works, granular mounds in three-dimensions have a lower angle of stability than two dimensional piles, and the approximately $5°$ difference in angles from Figures 3 and 6 can be explained by this. The same trend is also seen in Figures 4 and 7 with the repose angles, however, the data is not as consistent - most likely due to human error in data removal from the tapes.

The problems with planarity for these runs may have been a product of the shapes not being planar enough coupled with the drum's plexiglass bowing. When data from the initial experiments was looked at, there were pieces stuck, indicating that the pieces were not moving as freely and could thus stack higher, or that the drum was more planar and the pieces that were getting stuck no longer got stuck due to the plexiglass bowing. The differences in the spreads of dimers (which do not have the ability to be non-planer) show that the difference must have been the drum. Consistently, the spreads of doubles runs made prior to this had higher spreads than data found as is shown in Figure 9.

Upon this finding, the drum was reconstructed and the plexiglass sides were switched so what would have been convex was moved to the back and was then concave for both sides, resulting in a narrower space in the center than at the edges. Shims were added to make the edges even in tightness around the drum due to some minor differences in the aluminium. To ensure that this was not inducing artificial high data or low data from one region of the drum, test runs were measured to see if there were any patterns in the data for every 20 avalanches (one rotation).

Data was taken for mixtures of small and large triangles to determine if the ratio of particle size made a difference to the patterns of movement seen. However, in the course of one run (7-10 data points) there was a systematic decrease in the angles of stability, seen in Figure 10. However, there was no statistically significant difference in the repose angles and packing fractions for these runs, Figures 11 and 12 respectively.

To see if there was indeed a systematic trend in the data the data for mixtures of, by % small triangles 100, 0, and 50 were run again. These runs showed both a lower angle of stability and repose angle, shown in Figures 13 and 14. As expected, the difference for the 100% is greater than that of the other two redone as the 100% was the first data point taken. Further proof of a systematic deplanerizing of the drum is seen in the data for the densities of the data in Figure 15. Following the pattern for introduction of third dimensions, the greater the discrepancy in angle for Figures 13 and 14, the greater the packing fraction was for the redone data. In comparison to the data taken previously, the packing fraction had started at that level of planarity and decayed over the course of taking one data run.

To determine if the change in planarity was caused - or exacerbated - by filling the drum more than it had been in the past, data of 365g was retaken. The redone data was of the hexagons and singles mixture set, namely 100% hexagons and 90% hexagons 10% singles. Comparisons of the spreads were expected to be the same as the triangles and doubles-large diamonds runs; the results, however, were contradictory. The spreads in Figures 16 and 17 were of the same constancy (no additional outliers), however the change in average angle was not consistent. The averages, shown in Table 3, show non-uniform change in angle.

There was an additional way to check the planarity of the data taken by a histogram of the number of ball radii between each ball and the other balls. With this, the structures identifying planarity are found in the first two ball radii. For perfectly staked balls in two-dimensions there should be peaks at 1 and $\sqrt{3}$, given by perfect 30-6-90 triangle ratios, as at best ball bearings can be compacted at 60°. Data from runs of previously taken data and new data for doubles, 365g of hexagons, and 365g of 90% hexagons was used to see if there was a difference in the mass of ball bearing used on how planar the result was.

However, in all of the data the characteristic value of $\sqrt{3}$ was not seen, and 1 was seen in Figures 18-20. A possible cause of this is the ability of the center finding program to find the center of the ball with the precision needed for this task. One half-pixel can be the difference between 1 ball diameter and 1.5 ball diameters. Also of note is the characteristic peaks on each of the three figures. There is a consistent pattern of 3 peaks for the newer data and 2 peaks for the older data at the value of 1 ball diameter. Other images from the same data runs were compared and they also exhibit the same peak traits. A possible cause of this is the place of light reflection. Because the balls themselves are not flat inside the plane the place of reflection may differ across the drum from the angle of the camera. Another reason may be the particles' stacking in sheets in front of other sheets, or in the same sheet. If there is a third dimension added, particles may tessellate in planes angled "into" the drum that intersect with another plane angled "out of the drum" and also with planes "in line" with the drum.

There may be other complications being masked or contributing to the problems with planarity. The distinction between what is settling and what is an avalanche is not distinct enough. Through qualitative observation it may beneficial to add a criteria to the distinction of avalanche by pile stability. Multiple small, separate settlings may be equivalent to an entire avalanche if they take place within a short period of time/drum rotation. Determining why so many larger angled avalanches have a characteristic 'z' shape may help to clarify what is an is not maximally packed avalanches. It may also be of interest to determine how the 'z' shape originates from the other avalanches starting from a flat surface.

Other qualitative results for how particles move through the pile were studied while studying the critical angles and densities.

As previously stated, in this study it was also seen that small particles stay towards the center of the pile. An explanation for this is that smaller particles pack more densely than large particles. As the pile tilts the larger shapes move easier overtop the small particles than the small over the large, and the large particles end up at the bottom of the pile, along the edge. Small particles that move above the large particles get stuck in larger crevices and stay towards the center of the pile through that mechanism as well. Particle movement can be better seen by watching the data for coloured hexagons and doubles, or any other large image with different coloured markings mixed with smaller particles. From these studies it also appears that the larger shapes make circles around the nearly stationary center mass of small particles, or inert large particles. This effect seems to be greater the smaller the particle size ratio, Table 1.

Small amounts of small particles mixed with large particles appears to cause a dramatic change in stability due to imperfections in the lattice structure of large particles. By adding a small number of small shapes the ability of large shapes to bridge is severely compromised. This effect can be likened to stacking wooden children's blocks with a small ball stuck between two balls and then trying to tilt the tower upright; when the ball is present the entire block chain will pop out.

Conclusions

From this study it may be concluded that there is a noticeable difference in packing fraction and angle of stability for the introduction of a third dimension to a planar system. For this system, camera image quality has a noticeable effect on how images are processed, and thus on the understanding of angle of stability. The plexiglas of the original set up warps noticeably over the time period for one data set to be run. Histograms of the ball-diameter distance of the ball bearings show some insight to the structure of how the shapes are arranged.

Past runs should be re-run in the new drum design to ensure that this design is at least as planer as the data taken for previous data runs.

It may be useful to further study what constitutes an avalanche to be certain in the accuracy of angles and densities collected. A useful tool in this would be taking every third to fifth frame of one avalanche at various points in a run and comparing the movements of particles.

Other studies that may be fruitful are: study of past data taken's angle of repose to see if the same correlation occurs with the most stable characteristic angle for a mixture; looking at histogram data for past data sets to see if there is a pattern of the histograms due to mixtures. Qualitatively, the role of collapsing pores in lower 'stationary' portions of the mound should be looked at to explain the nature of avalanches.

References

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