

# Glauber Monte Carlo Study of 9GeV Au+Au Collisions at STAR

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## Abstract

This paper presents a Glauber Monte Carlo study of collisions between gold nuclei at  $\sqrt{s} = 9\text{GeV}$ . RHIC and STAR are briefly introduced. I then discuss the Glauber model, and the Monte Carlo and continuous approaches to implementing it. I discuss how I used the Glauber Model to fit a simulated RefMult curve to an empirical one, and present centrality cuts on experimental RefMult, impact parameter, number of participants, and number of collisions.

## 1 Introduction

The RHIC (Relativistic Heavy Ion Collider) at Brookhaven National Laboratory is a particle accelerator devoted to high-energy nuclear physics. There are two active detectors at RHIC, PHENIX (Pioneering High Energy Nuclear Interaction Experiment) and STAR (Solenoidal Tracker at RHIC). This study is based on data taken at STAR. Unlike other accelerators such as the LHC and Tevatron, RHIC's primary mode of operation is colliding heavy nuclei. Heavy nuclear collisions are of interest because they create a region of high energy density that contains many hadrons, allowing researchers to study the bulk properties of hadronic matter under extreme conditions. The primary subject of interest at RHIC is a state of matter called the Quark-Gluon plasma (QGP). In a QGP, hadrons break up into individual quarks and gluons. QGP is analogous to atomic plasma in that it is full of free color charges but is globally color-neutral (as is required by the principle of color confinement). In a QGP, interactions between quarks and gluons are weaker and simpler than in normal hadronic matter, so this state is a good environment for the study of the properties of individual quarks and gluons. In addition, it is believed that the universe was in the QGP state during the first few microseconds after the Big Bang, so studies of QGP could have applications for cosmology [1] (94).

Because QGP, as well as most of the particles produced during a heavy-ion collision, are so short-lived, it is necessary to study them indirectly by analyz-

ing the particles they emit as they evolve and decay. A major task for nuclear physicists is to create models that infer properties of the process of interest from the characteristics of particles observed by STAR's detectors. There are several important characteristics of every heavy-ion collision that are not directly observable. One number of interest is the impact parameter, or the distance between the centers of two colliding nuclei at closest approach. An impact parameter of zero corresponds to a head-on collision, and higher impact parameters correspond to more peripheral collisions. It is also desired to know the number of protons and neutrons (collectively, nucleons) in each nucleus that actually collide with one another during a certain nuclear collision. (Not all the nucleons in each nucleus interact during a typical heavy-ion collision. The ones that do are referred to as participants and the others are referred to as spectators). Another important parameter is the number of nucleon-nucleon collisions. This is not just half the number of participants because it is possible, and indeed common, for a participant in one nucleus to collide with several participants in the other nucleus [2] (118).

This paper is concerned with the relationship between those three parameters and the number of charged hadrons detected by STAR (called Reference Multiplicity or RefMult) during collisions between gold nuclei at center of mass energy of 9GeV. This study also divides impact parameter, number of participants, number of collisions, and RefMult into centrality bins, which give the range of each parameter that result from a certain, most central, percentage of nuclear collision events.

## 2 The Glauber Model

The backbone of this study is the Glauber model. This model is commonly used to simulate heavy-ion collisions. It uses either numerical or Monte Carlo integration to determine the number of participants and number of collisions for two nuclei of a certain type colliding with a certain energy at a certain impact parameter.

The Glauber Model assumes that the particle density of nucleons in a nucleus follow the Woods-Saxon density profile. This has the form:

$$\rho(r) = \frac{\rho_0}{1 + e^{\frac{r-a}{d}}} \quad (1)$$

where  $\rho(r)$  is the nucleon density,  $r$  is the distance from the center of the nucleus,  $a$  gives the approximate radius of the nucleus, and  $d$  is a measure of the skin depth, or how quickly the nuclear density falls off near the edge of the nucleus.  $\rho_0$  is fixed by the normalization condition:

$$\int \rho dV = \int_0^\infty 4\pi r^2 \rho(r) dr = A \quad (2)$$

Values used for these parameters were:  $a = 6.5$  fm,  $d = 0.535$  fm,  $\rho_0 = 0.16$  fm<sup>-3</sup>. [2] (113).

The Glauber Model also assumes that nucleons collide inelastically, and have some probability of producing observable particles on each collision. However, energy loss and change in momentum from each interaction are negligible, and so each nucleon can interact multiple times with the same cross-section. This static cross-section is assumed to be the same as the inelastic cross section for a single proton-proton collision; that is, the probability of two protons in different nuclei to collide does not depend on the nuclear environment. Finally, the Glauber model ignores the electromagnetic cross-section of proton-proton collisions and treats protons and neutrons as interchangeable.

There are two general ways of implementing the Glauber model. The first is to picture each nucleus as a continuous density distribution. Because nuclei are assumed to pass straight through each other, the two nuclei only “see” one another’s transverse density distribution. This is referred to as the thickness function, and it is obtained by collapsing the three-dimensional density distribution onto the transverse plane:

$$T(x, y) = \int \rho(x, y, z) dz \quad (3)$$

Here,  $z$  is the longitudinal direction and  $(x, y)$  make up the transverse plane, by convention [2] (118).

A collision of two nuclei at impact parameter  $b$  is modeled by the separation of two thickness functions by  $b$  in a transverse direction (taken to be the  $x$  direction, arbitrarily). The number of participating nucleons ( $N_{part}$ ) and the number of nucleon-nucleon collisions ( $N_{coll}$ ) are given by:

$$N_{part} = \iint \left( T(x, y)(1 - e^{-\sigma_{pp}T(x-b, y)}) + T(x-b, y)(1 - e^{-\sigma_{pp}T(x, y)}) \right) dx dy \quad (4)$$

$$N_{coll} = \sigma_{pp} \iint T(x, y)T(x-b, y) dx dy \quad (5)$$

where  $\sigma_{pp}$  is the inelastic cross-section for proton-proton collisions [2] (122-125). Its value is taken to be 31.5mb [5].

An alternate way to implement the Glauber model is to picture a nucleus as a bundle of  $A$  discrete nucleons. In this method, two nuclei are randomly populated with  $A$  nucleons each, fitting a Woods-Saxon density distribution. If  $X_1$ ,  $X_2$ , and  $X_3$  are three different random variables uniformly distributed on  $[0, 1]$ , then the nucleon location in spherical coordinates  $(r, \theta, \phi)$  is found using:

$$AX_1 = \int_0^r 4\pi r'^2 \rho(r') dr' \quad (6)$$

$$\theta = \arccos(1 - 2X_2) \quad (7)$$

$$\phi = 2\pi X_3 \quad (8)$$

These were obtained from D. Cebra during a consultation on this project.

One nucleus is translated by  $b$  in a transverse direction (again,  $x$  is used). The set of transverse distances  $r_{i,j}$  between every pair of nucleons from different nuclei is found, and a pair is assumed to have collided if the distance between them is less than the “nucleon radius” corresponding to the inelastic cross-section:

$$r_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \leq \sqrt{\frac{\sigma_{pp}}{\pi}} \quad (9)$$

$N_{part}$  and  $N_{coll}$  are then simply counted.  $N_{coll}$  is the number of nucleon-nucleon collisions, and  $N_{part}$  is the number of nucleons that undergo at least one collision. Because of the use of random numbers to do a numerical integration problem, this approach is classified as a Monte Carlo method.

The results of both methods are graphed below:

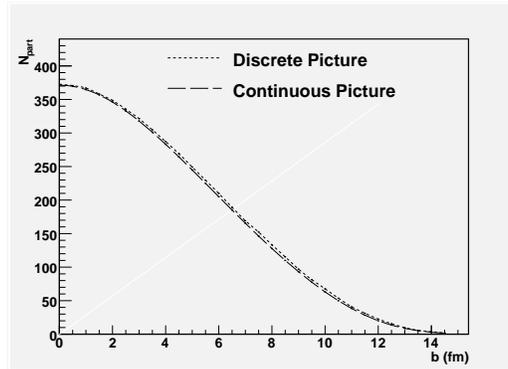


Figure 1:  $N_{part}(b)$ .

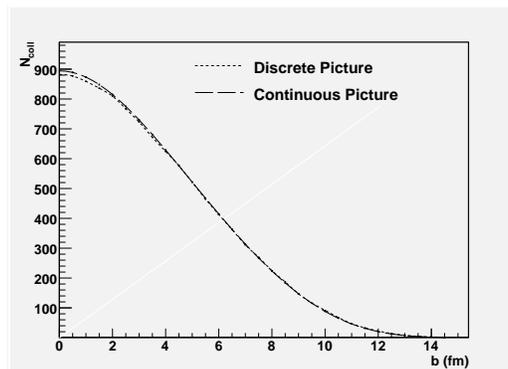


Figure 2:  $N_{coll}(b)$ .

Both techniques are numerical, so they are subject to errors. To find  $N_{part}$  and  $N_{coll}$  in the continuous picture, a two-dimensional Riemann sum was used. Upper and lower bounds for the correct value of the integral are given by the upper and lower Riemann sums. An approximate expected value is given by the average of the two, and the maximum error is given by the difference between the upper or lower and the average. In the discrete picture, it is assumed that a trial collision at a certain impact parameter will give results for  $N_{part}$  and  $N_{coll}$  that are normally distributed. Averaging over some number of results at the same impact parameter will give an unbiased estimator of the correct values for  $N_{part}$  and  $N_{coll}$ . Again, assuming normal distribution, the estimate of the error is given by  $E \sim \sigma/\sqrt{N}$ , where  $E$  is the error estimate,  $\sigma$  is the empirical standard deviation, and  $N$  is the number of trials. To summarize, the Riemann integral is made more accurate by shrinking the discrete intervals that are summed over, and the Monte Carlo method is made more accurate by increasing the number of trials averaged over. Interestingly, getting a certain precision took about the same amount of computer time for both methods.

### 3 Simulated RefMult distribution

To build a simulated RefMult distribution, I first simulated a typical run at STAR. To do this, I ran  $10^6$  Monte Carlo throws at random  $b$  with values on  $[0, 20]$  fm. The events were not averaged over; each event contributed to the RefMult distribution directly. Because of this, it was advantageous to use the Monte Carlo algorithm alone. The  $b$  values were weighted radially, because the fractional cross-section of a certain  $b$  goes as  $bdb$  [3]. I am confident that the  $[0, 20]$ fm interval covered all but a negligible fraction of the cross section because there were no nucleon-nucleon collisions with  $b > 19$  fm. All throws that resulted in no collisions were thrown out; this left about  $6 * 10^5$  live events.

I assume here that the RefMult produced by an event had a probability described by a Negative Binomial Distribution. This is a standard assumption in RefMult studies, for examples see [4] and [6]. A Negative Binomial Distribution has the form:

$$f(n; k, \mu) = \binom{n+k-1}{n} \frac{(\mu/k)^n}{(\mu/k+1)^{n+k+1}} \quad (10)$$

Where  $\mu$  is the mean, and  $k$  is a second parameter affecting the width. Sometimes an alternate parameterization is used:

$$f(n; k, p) = \binom{n+k-1}{n} p^k (1-p)^n \quad (11)$$

The relation between the two is:  $p = \frac{1}{\mu/k+1}$ , or  $\mu = k \frac{1-p}{p}$  [7].

Those two parameters  $\mu$  and  $k$  are related to  $N_{part}$  and  $N_{coll}$ . According to [6] (122), the relation is:

$$\mu = \alpha((1-x)\frac{N_{part}}{2} + xN_{coll}) \quad (12)$$

$$k = \beta((1-x)\frac{N_{part}}{2} + xN_{coll}) \quad (13)$$

In these equations,  $\alpha$  is the expected RefMult for a single proton-proton collision, and  $\beta$  is another fit parameter. The parameter  $x$  is not used in all parameterizations, but it is used in this one to determine what types of processes contribute to RefMult. It is assumed processes that contribute to RefMult can be divided into two parts. The first is the “soft” part, proportional to  $N_{part}$ . “Soft” processes are so called because they involve collisions at energies that are so low that the energy lost by the participants in particle production is enough to prevent them from colliding inelastically a second time and producing more particles. This is why the soft part of RefMult is assumed to depend only on  $N_{part}$ . The second is that “hard” part, proportional to  $N_{coll}$ . “Hard” processes are those that involve collisions that are so energetic that each participant can collide several times with no significant stopping from energy loss to particle production. This is why the hard part is assumed to depend only on  $N_{coll}$ . The fraction  $x$  indicates the relative contributions of hard and soft processes to the total RefMult [6] (122).

The fit parameters  $\alpha$ ,  $\beta$ , and  $x$  were fit using a least-squares optimization. It was assumed that  $x \leq 0.1$ , since  $x$  increases with center-of-mass energy, and  $x$  is found to be  $\sim 0.1$  for 130GeV in at least one study [6] (123). Unfortunately, when I ran a least-squares distribution, I got this:

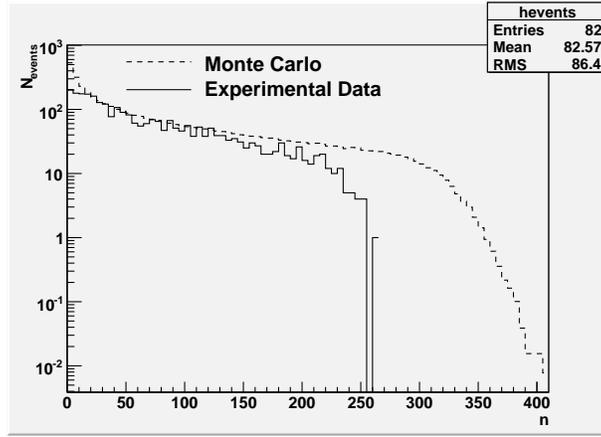


Figure 3:  $x = 0.1$ ,  $\alpha = 1.25$ ,  $\beta = 1.7$ . Fit is very poor.

I believe that this graph has that form because a least-squares distribution is biased toward fitting high values (which will have high absolute errors). It is unclear how to fix this without introducing new biases.

Next, I tried setting  $x = 0$ , and fitting  $\alpha = 1.1$  and  $\beta = 1.3$  by eye. This resulted in this graph:

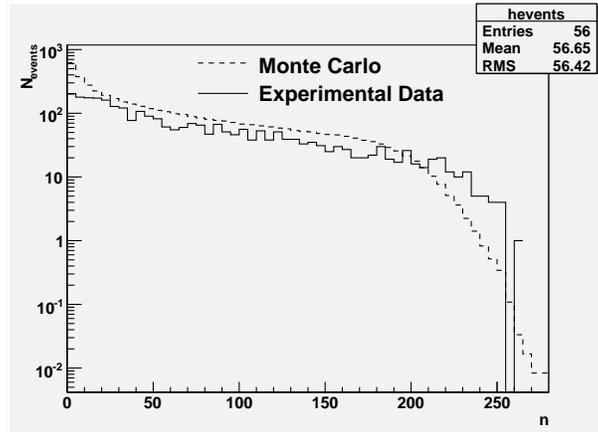


Figure 4: Second try at fitting RefMult

The above parameters resulted in these centrality cuts:

Centrality	Experimental RefMult	Calculated RefMult	$b$ (fm)	$N_{part}$	$N_{coll}$
0.1	265	195	5	325	680
0.2	165	155	7	265	530
0.3	125	130	8.5	220	410
0.4	95	105	9.5	180	320
0.5	75	90	10.5	150	245
0.6	50	70	11.5	120	180
0.7	35	60	12.5	95	135
0.8	25	45	13.5	75	95
0.9	15	35	14.5	60	65
1.0	10	30	19	45	45

## 4 Conclusions and Future Work

The first part of this project, implementing the Glauber Monte Carlo method, works very well. However, the second part, using the Glauber data to fit RefMult, has not succeeded. Fitting by eye works reasonably well, but I have not found a good, systematic way to get a fit. Once that is done, accurate centrality cuts can be created. Also, a good fit can be used to give an estimate of the trigger inefficiency of a particular RefMult data set, by taking the ratio between the predicted and empirical distributions at low RefMult.

## 5 Acknowledgements

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