The Relativistic Collisions of Au+Au: *A Theoretical Approach and Analysis*

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Abstract

A theoretical analysis on relativistic heavy ion collisions is presented. The Glauber Model is studied through a comparison to the Hard Sphere Approximation– a simpler model, using their calculations for participant multiplicity. The Glauber Model is further studied through a comparison between its two different versions, the Monte Carlo Glauber and the Optical Glauber, using their calculations for participant and collision multiplicity from both of its versions, the Monte Carlo Glauber and the Optical Glauber. The theoretical simulation generated from the Optical Glauber's calculations for participant multiplicity is also discussed. This simulation is then used in the presentation and analysis of the $\sqrt{s_{NN}}$ =19.6 GeV data set at RHIC using the STAR detector.

I Introduction

The study of relativistic heavy ion collisions relates to both Nuclear and Particle Physics, and it deals with the behavior of heavy ions when they collide after being accelerated to approximately the speed of light. Currently, the fastest speed used in these collisions is 0.99995c, where c is 3×10^8. Experiments are conducted at laboratories that have particle accelerators and detectors. After several runs, data is then collected and analyzed using computer programs. The central goal of the UC Davis division of Nuclear Physics is to study heavy ion collisions in order to determine the existence of Quark Gluon Plasma (QGP), a new state of nuclear matter. They receive a majority of their experimental data from the Solenoid Tracker at RHIC (STAR), which is located at the Brookhaven National Laboratory in New York.

In this analysis, a theoretical approach will be taken on heavy ion collisions by studying the calculations from the Glauber Model. This approach involves obtaining a deeper understanding of the Glauber Model's complexities and creating a simula-

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tion that relates its theoretical calculations for participant multiplicity to experimental data. The relevance of finding a theoretical simulation is that it serves as a basis for data analysis. With the use of a generated simulation, the data set from the $\sqrt{s_{NN}}$ =19.6 GeV run at RHIC using the STAR detector will be also studied in this analysis. The importance of the final portion of this analysis is that it addresses the presumed errors in charged particle multiplicity and the trigger inefficiency in the 19.6 GeV data set, and it allows corrections to be made before the results are published.

II Theory

A Introduction to Heavy Ion Collisions

In relativistic heavy ion collisions, a collision between two nuclei is referred to as an event. The two main types of events include central, which is where the nucleons hit straight on, and peripheral, which is where the two nuclei graze each other during the event. The degree of centrality is determined by the magnitude of the impact parameter,which is the distance between the center of two nuclei. The number of participants during a single event refers to how many nucleons from one nucleus collide with other nucleons in another nucleus. The number of collisions during a single event refers to all the nucleon-nucleon interactions between participants.

B The Number of Collisions and Participants

A very rough estimate for the number of participants in a single event can be obtained using the Hard Sphere Approximation (HSA). The HSA depicts the nucleus as having a definite radius, R, which means that no nucleons exist outside its radius. Within this approximation, the nucleon density is constant throughout the entire nucleus,

$$\rho(r) = \frac{A}{\frac{4}{3}R^3} \approx \rho_0$$

where A is the number of nucleons in the nucleus. A rough estimate of the number of participants can be found simply by finding the overlapping volume between the intersection of the two nuclei and multiplying this volume by the nucleon density,ρ_0. The amount of overlapping volume is determined by the magnitude of the impact parameter.

The difference between the participant multiplicity found using the HSA as compared to a more realistic Glauber Model is rather small until the most peripheral events are considered [3]. Since the HSA depicts a nucleus that does not have nucleons outside its nuclear radius, this model cannot offer an explanation for how there are still participants and collisions at impact parameters greater than twice the nuclear radius. This approximation is also unable to provide a way to estimate the total number of collisions that occur during an event. The Glauber Model provides a much more realistic depiction of the nucleus and offers mathematical methods that can more accurately calculate values for the participant and collision multiplicity during a single event. It is currently the most accepted model in heavy ion collision research.

Within the Glauber Model, the nucleus is no longer depicted as having a definite, definable radius. Instead, the model portrays the nucleus as having nucleons tightly compacted in its center, which trail off into an infinitely long, thin tail beyond the mean radius. The density is no longer constant in the Glauber Model, and it is parameterized by the Woods-Saxon parameterization,

$$\rho(r) = \frac{\rho_0(1+\omega(\frac{r^2}{c}))}{exp(\frac{r-c}{z})+1}$$

where ρ_0 is the initial nucleon density,$c \approx 1.07A^{\frac{1}{3}}$, and z is related to the nucleus's skin thickness. Within Figure 1, the respective nucleon densities for Au of both the Hard Sphere Approximation and the Glauber Model are shown. Within the Optical Glauber Model, the number of collisions at a given impact parameter is based on three different factors. The first factor involves the probability of finding a nucleon in the overlapping volume of nucleus A. The probability for finding one of the A nucleons is proportional to,

$$P_A = \frac{1}{A}\int \rho(\overrightarrow{b_A}, z_A)dz_A d^2b_A =$$

$$\frac{1}{A}\int T_A(b)d^2b_A$$

where,

$$T_A(b) = \int \rho(\overrightarrow{b_A}, z_A)dz_A$$

is the thickness of A. The second factor involves the probability of finding a nucleon in the overlapping volume of B. The probability for for finding one of these B nucleons is proportional to,

$$P_B = \frac{1}{B}\int \rho(\overrightarrow{b_B}, z_B)dz_B d^2b_B =$$

$$\frac{1}{B}\int T_B(b)d^2b_B$$

where,

$$T_B(b) = \int \rho(\overrightarrow{b_B}, z_B)dz_B$$

is the thickness of B. The final factor involved in the probability of a nucleon-nucleon interaction involves the cross-section for an inelastic collision, which is where the nucleons interact with each other instead of passing by each other unaffected. One of the most simplest expressions for this probability is,

$$\sigma_{inelas}\delta(\overrightarrow{b} - \overrightarrow{b_A} - \overrightarrow{b_B})$$

where σ_{inelas} is the magnitude of the inelastic cross-section [3]. This cross-section is determined by $\sqrt{s_{NN}}$, the initial energy of the nuclei in the event, and is 42 ± 1 when $\sqrt{s_{NN}}$ is equal to 200 and 130 GeV [2]. After incorporating all these facotors,

$$P_{AB}(b) = \frac{\sigma_{inelas}}{AB}T_{AB}(b)$$

where,

$$T_{AB}(b) = \int d^2sT_A(s)T_B(|\overrightarrow{b} - \overrightarrow{s}|)$$

and s is the distance from the center of nucleus A to any point within nucleus B. The number of collisions during an event is related to this probability by,

$$N_{coll} = (AB)P_{AB} = \sigma_{inelas}T_{AB}(b)$$

The number of participants is related to the number of collisions such that it is based on the probability for a nucleon from A to collide with each of the nucleons in B. The probability for a nucleon,h, from A to have n collisions,

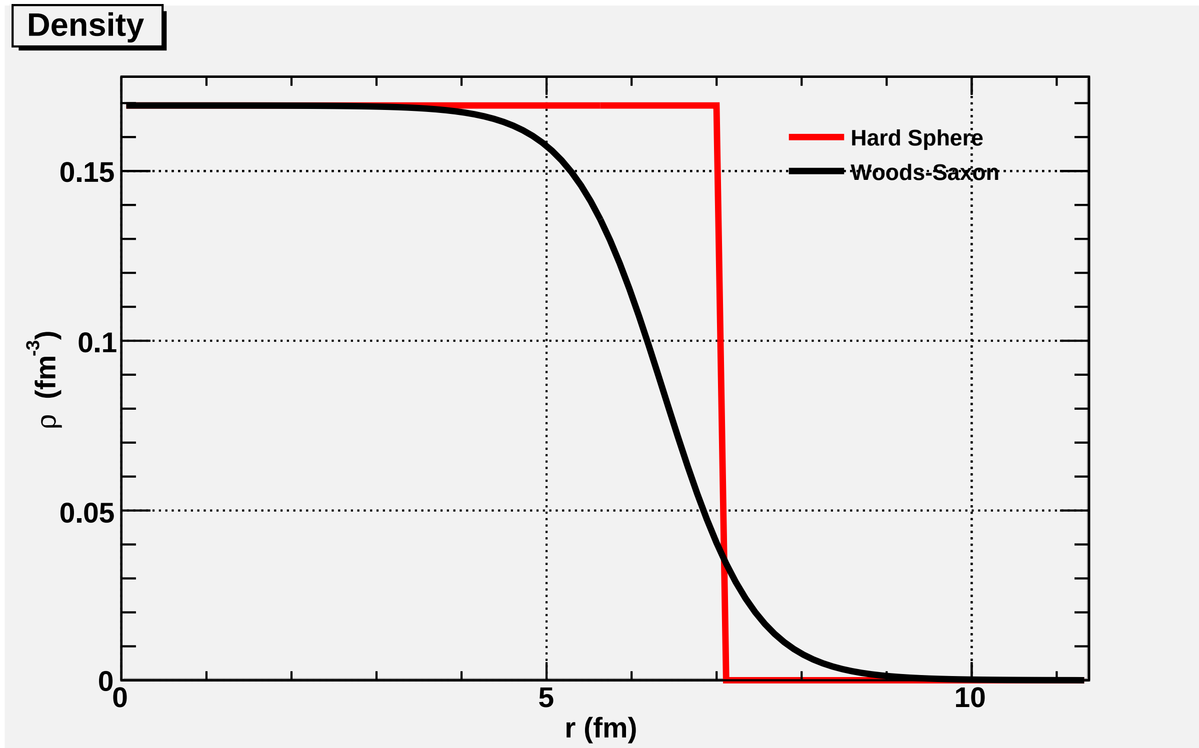

Figure 1: This is a plot of the densities of Au for both the Woods-Saxon and Hard Sphere Approximation as a function of r.

$$P_{Ah}(n,b) = \binom{A}{n}\left[1 - \frac{\sigma_{inelas}}{A}T_A(b)\right]^{A-n}$$

where,

$$\binom{A}{n} = \frac{A}{(A-n)!n!}$$

The total number of participants in A and B can be found by integrating over A's overlapping cross-sectional area, weighing it by the sum of probabilities for a nucleon from A interacting with the nucleons in B, and doing the same for B, which will produce the expression,

$$N_{part}(b) =$$

$$\int d^2s[T_A(s)(1 - exp(\sigma_{inelas}T_B(|\overrightarrow{b} - \overrightarrow{s}|))$$

$$+(T_B(|\overrightarrow{b} - \overrightarrow{s}|)(1 - exp(\sigma_{inelas}T_A(b))]$$

where,

$$\left[1 - \frac{\sigma_{inelas}T_A(b)}{A}\right]^A \approx exp(-\sigma_{inelas}T_A(b))$$

if $\frac{\sigma_{inelas}T_A(b)}{A} << 1$.

The one problem with the Optical Glauber Model is that it offers a slightly inaccurate portrayal of the nucleus. This is because it eliminates all randomness by treating the nucleons as a continuous fluid, which are fixed in space for every nucleus. The Monte Carlo version of the Glauber Model resolves this problem by randomly populating the nucleus with nucleons according to the Woods-Saxon nucleon density function. If the cross-sectional area between a nucleon in A and a nucleon in B is equal to the inelastic cross-section, or in terms of the radial distance between the two nucleons,

$$r_{AB} = \sqrt{\frac{\sigma_{inelas}}{\pi}}$$

then the nucleon in A will interact with the nucleon in B; both will be participants. If this is done for each nucleon in both nuclei, the number of collisions and participants can be obtained for one possible nucleon population in A and B. This process is usually repeated in order to take the average number of participants and collisions at a given impact parameter.

C Relating Theory to Experiment

One of the problems with finding the number of participants and collisions at a given impact parameter is that these calculations cannot immediately be related to theory. The Glauber Model can only calculate distinct values for the number of participants and collisions for a single event. These calcuations also do not include any fluctuations. What is even more problematic is that the number of participants and collisions are two quantities that cannot even be experimentally measured.

However, these issues can be resolved by correlating the theoretical calculations of participant and collision multiplicity to the following two experimentally measureable quanties: charged particle multiplicity, which refers to the number of charged particles produced during an event, and how many events occurred per a given multiplicity.

Without including fluctuations in the theoretical calculations, it is assumed that only a certain number of collisions and participants can occur at a given impact parameter. To incorporate statistical variation in these quantities, the number of events is redistributed in a Poisson distribution where,

$$\langle N_{part} \rangle = \sqrt{N_{theopart}}$$

and

$$\sigma_{dev} = \sqrt{N_{theopart}}$$

In order to predict the probability of having a given amount of participants and collisions the geometric cross-section is considered. This cross-section can be expressed as,

$$\frac{d\sigma}{db} = 2\pi \int b(1 - exp(-\sigma_{inelas} T_{AB}(b)))d^2b$$

which is shown in Figure 2. As a result of the Woods-Saxon parameterization, the cross-section slowly tails off to zero instead of sharply dropping off to zero when $b \approx 2R$, where R is the mean nuclear radius. The probability for having a given multiplicity is proportional to,

$$\frac{d\sigma}{dN_{coll}} \; or \; \frac{d\sigma}{dN_{part}}$$

which both can be found by applying the chain rule to the preceding equation. Each of the Poisson distributions are then weighed by their respective $\frac{d\sigma}{dN_{part}}$ or $\frac{d\sigma}{dN_{coll}}$ and added together to create a continuous distribution of the number of events versus participant or collision multiplicity.

The final step in relating theory to experiment involves a conversion between the number of participants or collisions and the number of charged particles experimentally received. This conversion is achieved through a correlation between the mean number of charged particles and the mean

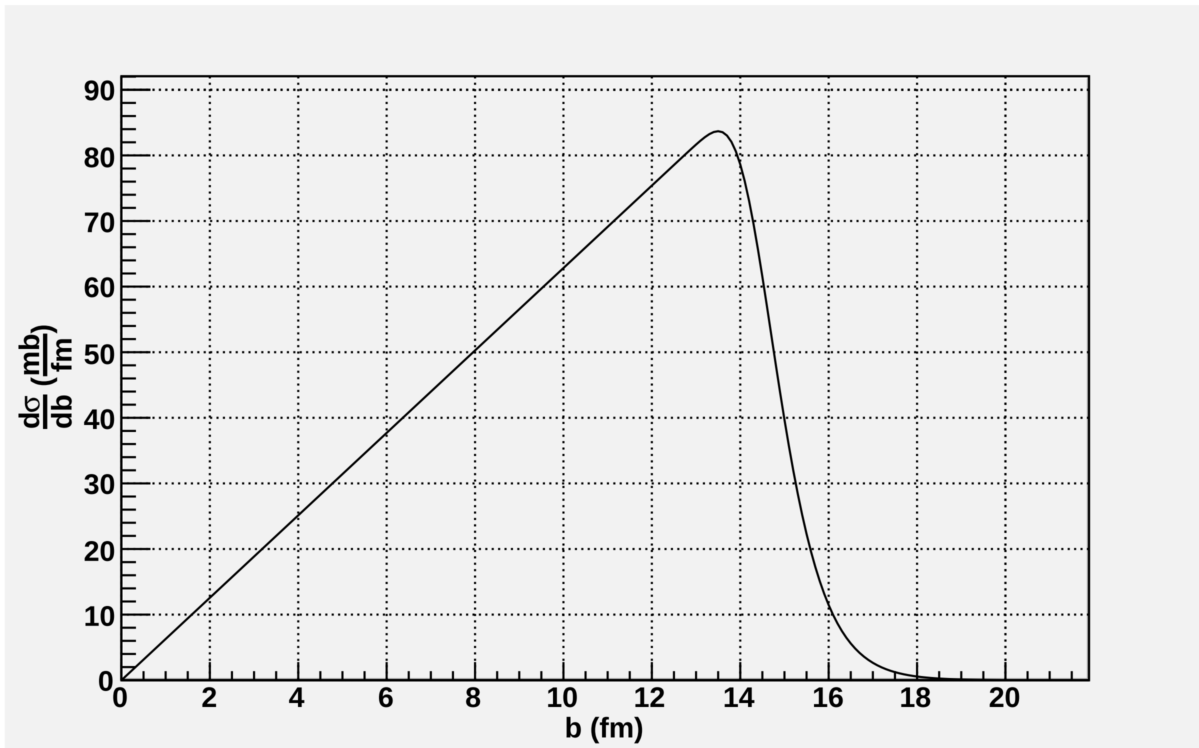

Figure 2: This is a plot of the geometric cross-section -vs-b for Au. As the impact parameter increases, the probability of having an event increases.

Npart or Ncoll. An additional 0.87 prefactor must also be considered in the experimental data due to tracking biases [2]. Once this conversion factor is acquired, each N_{part} or N_{coll} is then multiplied by this constant, which creates a number of events versus charge particle multiplicity distribution.

D Application of Theory

Finding a model that agrees with experimental data can be extremely useful because it can be used later to analyze experimental results through comparison. A prime example of where theory could be used to analyze data from relativistic heavy ion collisions is in determining trigger detector efficiencies. At STAR, the two triggers are the Central Trigger Barrel (CTB), which records charged particle multiplicity, and the Zero Degree Calorimeter (ZDC), which records the number of neutrons that are spectators in an event. The efficiency of these trigger detectors refers to how well they are able to function during a run. In order to approximate the collective efficiency of both the CTB and the ZDC, a simple ratio can be taken,

$$\epsilon_{trigger} = \frac{N_{chmeasured}}{N_{chtheory}}$$

where the numerator is the experimental multiplicity and the denominator is the theoretical multiplicity. If the experimental multiplicity is truly a subset of the theoretical multiplicity, then this ratio will range from zero to one. However, if the ratio is simply a comparison between two independent sets, then the range of the ratio is no longer restricted to being less than or equal to one.

III Techniques

A Computer Programming

In relativistic heavy ion collisions, computer programming has become the basis for both experimental and theoretical research. While each experimental run yields a large amount of data that needs to be analyzed, manipulated, and plotted, theoretical calculations require additional tasks such as integrating complicated functions over obscure regions. Although there are no restrictions on what language to use when writing programs for research in relativistic heavy ion collisions, most programs are written in C++. This preference has highly been influenced by the popularity of Root, an object-oriented program specifically designed for data analysis, which can only interpret C++ command language.

Regardless of what language is used in an algorithm, numerical integration is one of the most fundamental capabilities a program designed for theoretical calculations must have. Regardless of whether any of the involved theoretical expressions can be integrated by hand, it is much easier to create a computer program that can numerical integrate these expressions. Although numerical integration must be over a definite region and does not provide a function to the integral, it works extremely well when only numerical values are desired. In C++, numerical integration can be written using `for` loops and increasing each recursive variable by a desired width, dx . For most of the programs written for this project, the incremental width was 0.5 fm, which relates to the smallest amount that can currently be measured. Multivariable numerical integration in C++ is simply achieved by embedding `for` loops into other `for` loops.

As mentioned before, one problem involved in creating a program that uses numerical integration to carry out calculations is that it cannot provide a function to the integral, which can then be inverted in order to find the respective independent variables for evenly spaced dependent variables. However, this issue can be worked around by using very small incremental widths in the `for`loops and utilizing the `if` command to store only the independent variables that produce the evenly spaced dependent variables. This somewhat time-consuming technique was only used for this analysis when obtaining the respective impact parameters of evenly spaced values of N_{part} and N_{coll}.

Another main capability that is necessary for carrying out theoretical calculations is being able to distribute numbers randomly according to a given distribution. Although such a capability can definitely be written from scratch, Root has built in functions that can redistribute numbers according to either built-in probability distributions or user-defined functions. While the Poisson function of the Trandom, pseudo-random number generating, class was used to redistribution the participant multiplicity, the GetRandom3 function of the TF3, three-parameter function, class was used to randomly populate the nucleus for the Monte Carlo Glauber Model. Although both of these functions can only generate one random number at a time, they can generate the desired amount of random numbers when embedded into a `for` loop.

IV Results

A Comparing Models

In order to develop a deeper understanding of the Glauber Model, the Optical Glauber Model and Hard Sphere Approximation calculations for participant multiplicity were compared. When comparing these values, it became quite apparent which regions the accuracy of the Glauber Model affects the most. These two main regions occurred in the most central and peripheral regions, as seen in Figure 3. This agreed with what had been predicted before the calculations. The reason why the HSA's average participant multiplicity was so large at $b = 0$ relative to the Optical Glauber was because it is where the assumption that every nucleon within the region of nuclear overlap will participate was affected the most. The HSA's average number of participants immediately drops off to zero when the impact parameter is approximately twice the average radius, instead of slowly tailing off as in the Optical Glauber curve. This was due to the HSA's assumption of a definite nuclear radius. Therefore, the Glauber Model's incorporation of the probability of having a nucleon-nucleon eliminates the extreme calculations found using the HSA.

Surprisingly, the Monte Carlo Glauber calculations for participant and collision multiplicity were not as close to the Optical Glauber's same calculations. The participant multiplicity found using the Monte Carlo Glauber was consistently higher than the participant multiplicity found using the Optical Glauber. The ratio between the former and the latter also increasingly diverged from unity as the impact parameter increased,which can be further observed in Figure 4 and Figure 5. Although the rapid fluctuations of the ratio at $b \approx 14$ fm are to be expected because the probability of a collision is extremely unlikely in such peripheral events, the ratio for more central events should have been closer to one.

There was also disagreement between Monte Carlo's and Optical's calculations for collision multiplicity. While the Monte Carlo's calculations for participant multiplicity were always higher than the Optical's calculations, such consistent behavior did not occur for collision multiplicity. Although the Optical's calculations for the number of collisions were higher for the most central events, the MC's calculations eventually became larger as the impact parameter increased, seen in Figure 6. Except for the most central events, the ratio between the MC and the Optical collision multiplicity was also greater than unity, as in Figure 7.

The lack of agreement between the two versions of the Glauber Model occurred most likely because of the Monte Carlo's random placement of the nucleons far away from the center. This would have caused the bulk of the nucleon density in center to be pushed out, which could have never occurred in the Optical's fixed nucleons, which are treated as being part of a continuous liquid that thins out as it leaves the centers. The Monte Carlo's nucleon density push would result in more particles being at some distance away from the center, which would cause a higher number of participants at large impact parameters than the Optical anticipates. The presence of nucleon bulk at a distance away from the center would also affect the number collisions in the following two ways. One effect would be that less collisions could occur in the most central events, or events in the $0-30\%$ centrality regions likewise, than the Optical anticipates

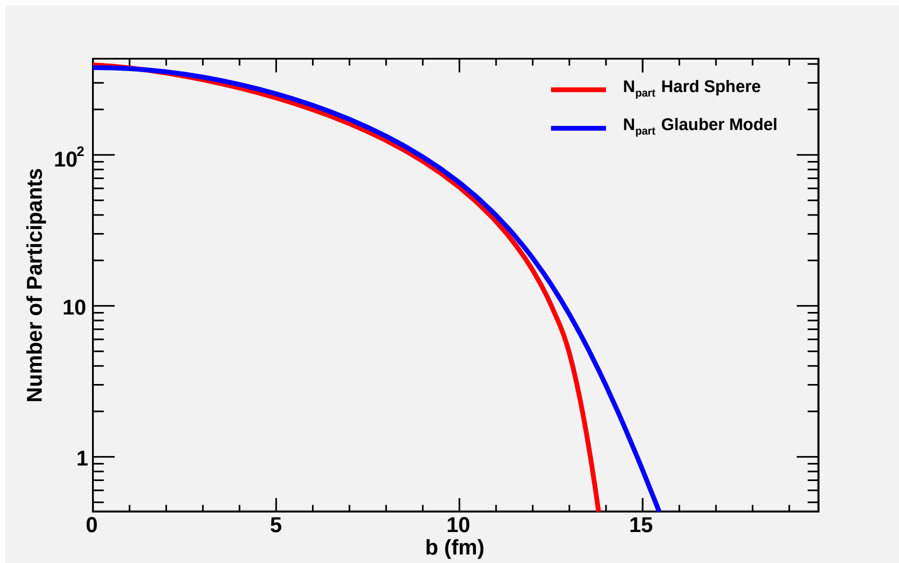

Figure 3: This is a plot of the participant multiplicity found using the Hard Sphere Approximation and the Optical Glauber Model.

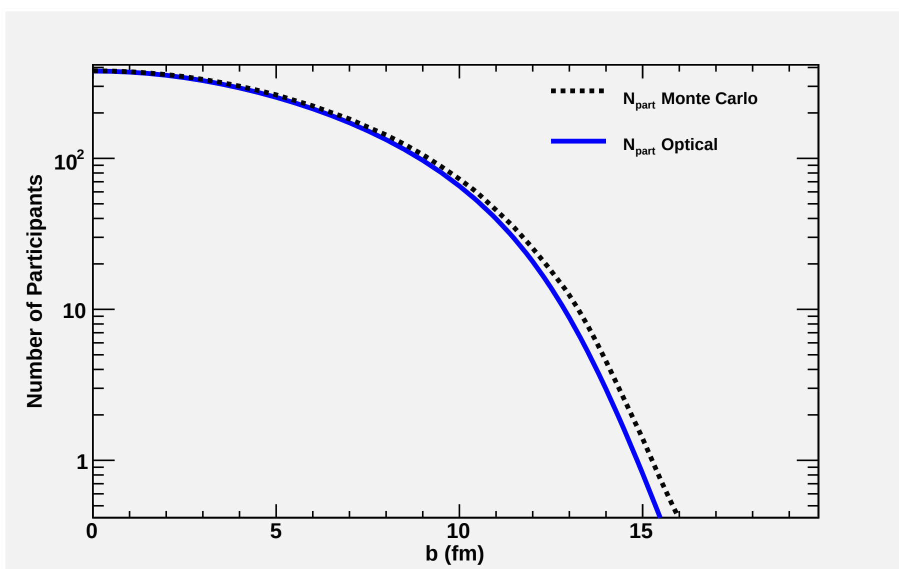

Figure 4: This is a plot of the participant multiplicity found using the Optical Glauber and the Monte Carlo Glauber.

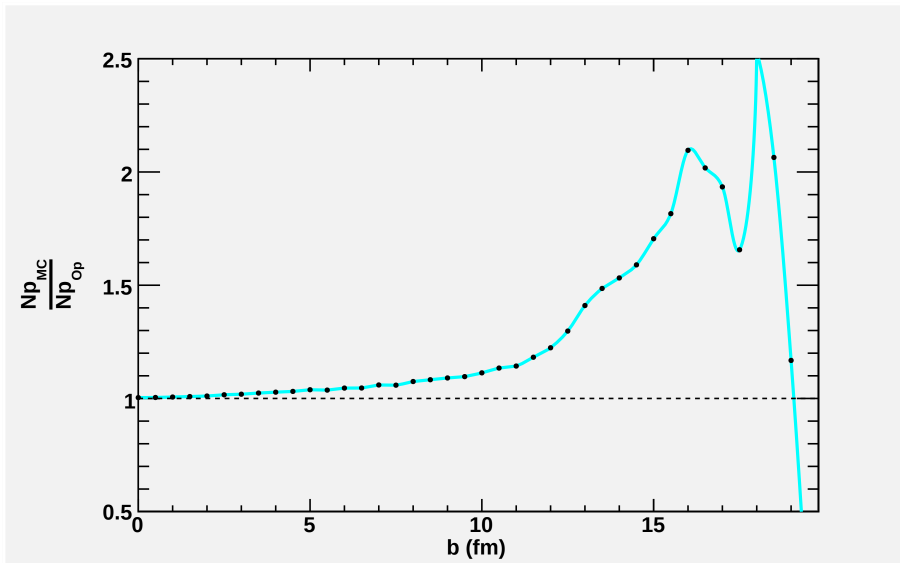

Figure 5: This is a plot of the ratio of the Monte Carlo over the Optical participant multiplicity. Observe how this ratio is greater than one at all impact parameters.

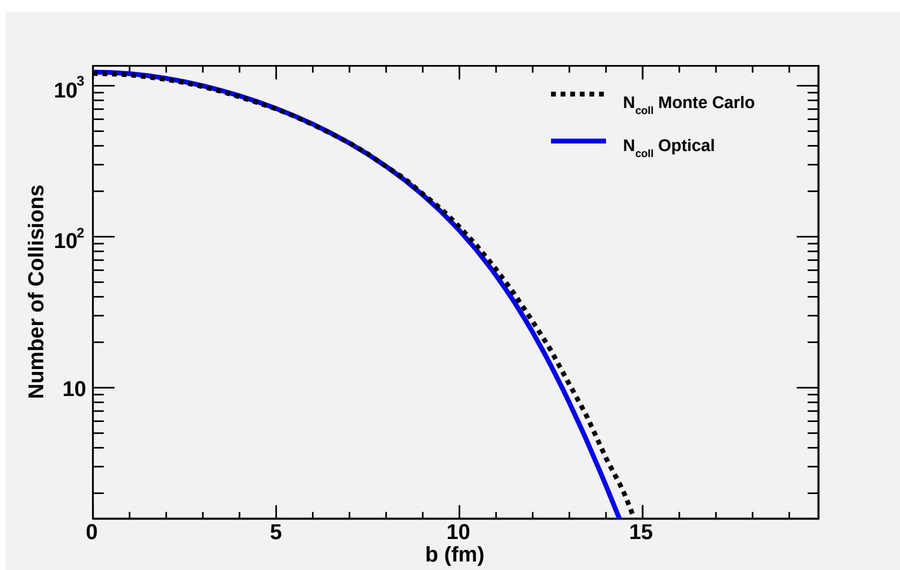

Figure 6: This is a plot of the collision multiplicity found using the Optical Glauber and the Monte Carlo Glauber.

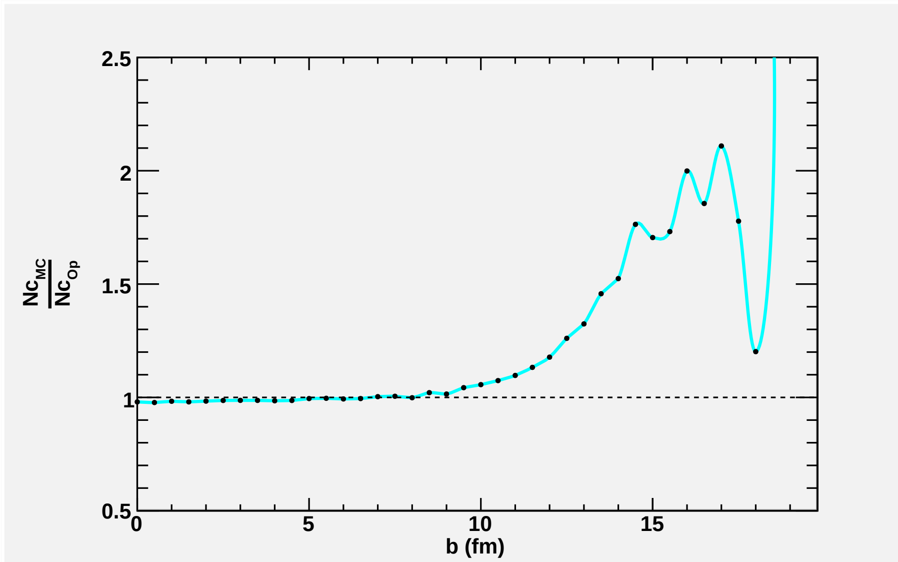

Figure 7: This is a plot of the ratio of the Monte Carlo over the Optical collision multiplicity. Observe how this ratio is greater than one at most impact parameters.

because less nucleons are populated around the center. Another effect would be that more collisions would occur in more peripheral events, or events in the $30-100\%$ centrality regions, because more nucleons are away from the nucleus. A summary of these results can be further observed in Table 1 and Table 2.

B Glauber Simulation

To generate a Glauber Simulation of numerous events, the Optical Glauber calculations were chosen instead of the Monte Carlo Glauber calculations. This was because no model was more accurate than the other, and it was much more efficient to calculate even-spaced number of participants using the Optical Glauber Model. The final generated curve, which was generated by manipulating the theoretical calculations for participant multiplicity, strongly resembled the curve generated using experimental data, Figure 8. As expected, the number of events for lower multiplicities is higher than the number of events for higher multiplicities because the events with large impact parameters were more probable. Although the theoretical simulation curve peaked higher than the raw data around $Nch \approx 8$, the curves aligned quite well at larger multiplicities.

C Trigger Efficiency

In order to analyze the trigger efficiency at 19.6 GeV, the raw data sets from the 200 GeV and 130 GeV runs were scaled down by eye to match up with the 19.6 GeV raw data set as much as possible; the same scaling procedure was done to the Glauber Curve. Although each of the raw data sets aligned quite well for the $0-10\%$ and $30-70\%$ centrality regions, the 19.6 GeV data diverged greatly in the $10-30\%$ and the $70-100\%$ centrality regions, as depicted in Fig. (8). The intensity of this divergent behavior was made even clearer after the ratios between

$\frac{d\sigma}{db}$	$\langle Np_{OP} \rangle$	$\langle Np_{MC} \rangle$
$0-10\%$	331.78	337.03
$10-30\%$	225.07	236.02
$30-50\%$	97.09	106.43
$50-70\%$	34.58	41.19
$70-100\%$	2.43	3.44

Table 1: Table on the mean participant multiplicity based on the percentage of the $\frac{d\sigma}{db}$ distribution.

$\frac{d\sigma}{db}$	$\langle Nc_{OP} \rangle$	$\langle Nc_{MC} \rangle$
$0-10\%$	1025.27	1008.86
$10-30\%$	573.13	571.53
$30-50\%$	185.21	190.40
$50-70\%$	43.16	49.17
$70-100\%$	2.19	2.97

Table 2: Table on the mean collision multiplicity based on the percentage of the $\frac{d\sigma}{db}$ distribution.

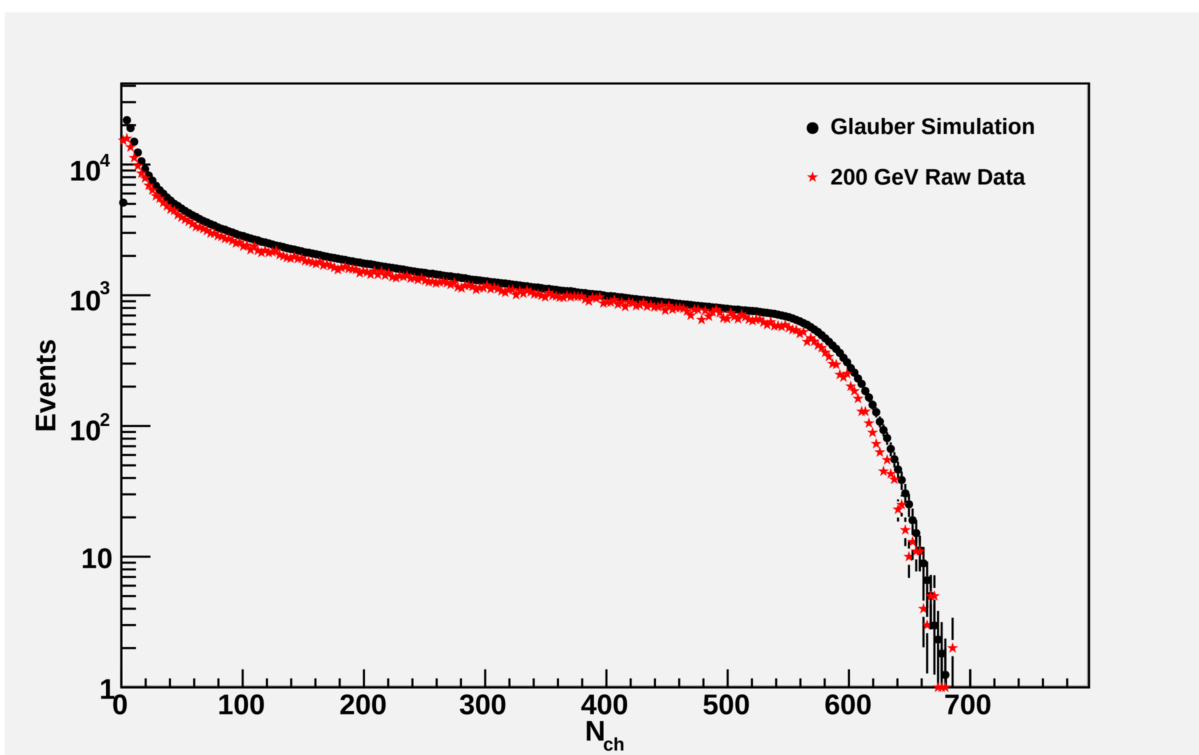

Figure 8: This is a plot of the Glauber Simulation and a set of raw data from a $\sqrt{s_{NN}} = 200\ GeV$.

each of the data sets and the Glauber Simulation were taken, which is shown in Figure 10. While the efficiency curves for the 130 GeV and 200 GeV runs were centered around unity for most multiplicities, the efficiency curve for the 19.6 GeV curve dips greatly from unity in $10-30\%$ and $70-100\%$ regions of centrality.

The observed inefficiency during the 19.6 GeV run has been determined to be caused by the zero degree calorimeters (ZDC) [1]. In higher energy collisions, such as $\sqrt{s_{NN}}$ = 200GeV or 130GeV, the neutral spectators in an event have enough longitudinal momentum to prevent their opening angle from extending the angle covered by the ZDC. However, the insufficient amount of longitudinal momentum in the $\sqrt{s_{NN}}$ =19.6 GeV allowed the spectators to have a larger opening angle than the ZDC coverage. This was the main cause for the inefficiency for events with impact parameters of $\approx$ 8 fm or less. The only reason why the most central events, or the events within the $0-10\%$ centrality bins likewise, were not affected by this inefficiency is because the efficiency of the CTB was able to compensate for the ZDC at these impact parameters. The divergence during the most peripheral events, which were the events within the $70-10\%$ centrality bins, occurred because the fragments of nuclear matter did not break up into protons and neutrons after a collision. Instead, the spectating neutrons were bound into charged particles, such as alphas and deuterons, and deflected from the ZDC by the DX magnet, which lies in front of the ZDC [1].

Despite the extremely divergent behavior of the $\sqrt{s_{NN}}$ =19.6 GeV data set within the $10-30\%$ and $70-100\%$ centrality bins, the mean number of charged particles across these bins was no more or less than a unit from the simulation curve, as shown in Table 3. The ranges differed slightly from the table in [1]because the centrality regions were based on the theoretical simulation curves's cross-section instead of the cross-section from $\sqrt{s_{NN}}$ =130 GeV. However, if the same cutsfor the centrality bins as in [1], then the mean numnber of both charged particles and participants was still no more or less than a unit from the simulation curve, as shown in Table 4. These results ultimately showed how the experimental data was not largely affected by the trigger efficiency as presumed before this analysis.

V Conclusion

Within this analysis, the theoretical calculations from the Hard Sphere Approximation and the two different versions of the Glauber Model were reported. The comparison between the calculated values for participant and/or collision multiplicity portrayed most of the complexities within the Glauber Model. The resultant simulation generated using the the Optical Glauber's calculations for participant multiplicity was analyzed and discussed. From the comparison between the simulation and the raw data from a $\sqrt{s_{NN}}$=200 GeV, the Glauber Model seems even more remarkable than before. Not only can its mathematical expressions be manipulated to predict what occurs experimentally, but the theory behind the model provides an intuitive idea of what physically occurs during an event. The results from the comparison between the simulation and the three raw data sets were also presented and discussed, in addition to each of their respective trigger efficiencies. From these results, it can be concluded that the trigger efficiency of the $\sqrt{s_{NN}}$ =200 GeV

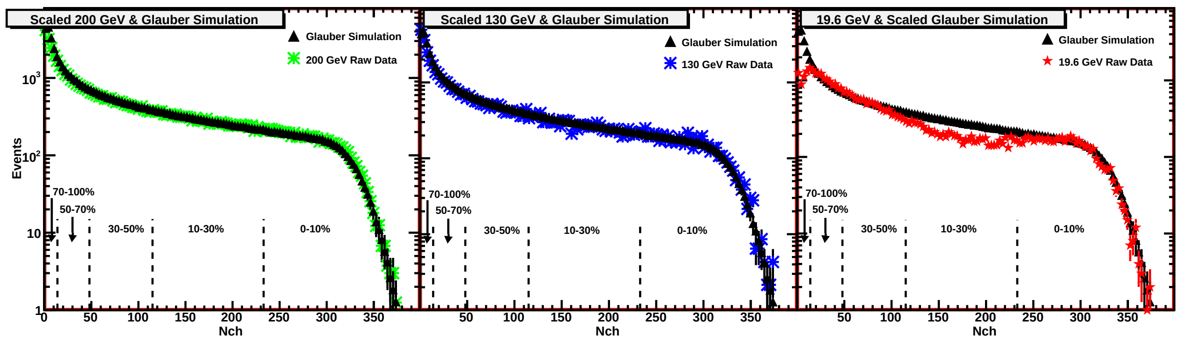

Figure 9: This is a plot of the number of events from each data set. Observe how only the 19.6 GeV data set exhibits divergence from the Glauber Model predictions.

$Centrality$	$Nch\ Range$	$\langle Nch_{19.6GeV} \rangle$	$\langle Nch_{Glauber} \rangle$
$0-10\%$	$233-374$	280.31	279.38
$10-30\%$	$115-233$	167.21	168.10
$30-50\%$	$48-115$	76.51	77.99
$50-70\%$	$14-48$	29.03	28.18
$70-100\%$	$0-14$	7.86	6.38

Table 3: Mean N_{ch} based on centrality of the cross-section of the Scaled Glauber.

$Centrality$	$Nch\ Range$	$\langle Nch_{\mathrm{19.6GeV}} \rangle$	$\langle Nch_{\mathrm{Glauber}} \rangle$	$\langle Np_{\mathrm{19.6GeV}} \rangle$	$\langle Np_{\mathrm{Glauber}} \rangle$
$0-10\%$	$237-374$	282.36	281.74	185.85	185.47
$10-30\%$	$117-233$	170.323	170.97	112.13	112.56
$30-50\%$	$48-115$	77.21	78.82	50.83	51.89
$50-70\%$	$14-48$	29.03	28.18	19.11	18.55
$70-100\%$	$0-14$	7.86	6.38	5.17	4.20

Table 4: Mean N_c and N_p based on the cross-sectional cuts taken in [1].

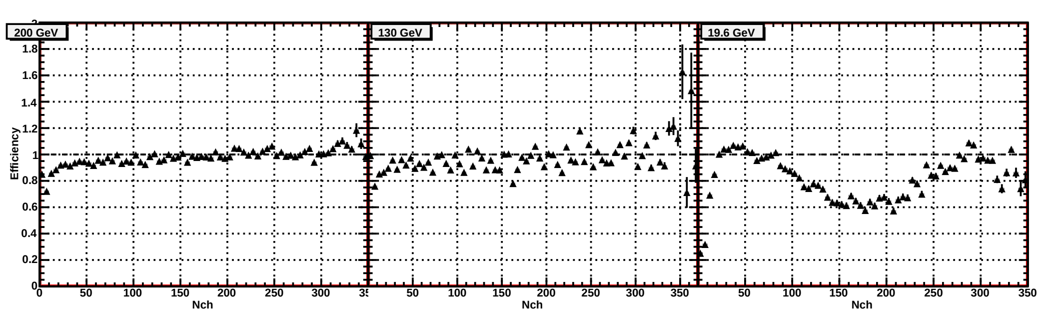

Figure 10: This is a plot of the efficiencies from each data set. Observe how only the 19.6 GeV data set exhibits divergence from one.

data did not really affect the charged particle multiplicity as much as what had been anticipated.

In one aspect, the objective of this theoretical approach has been achieved, such that a deep enough understanding of the Glauber Model was obtained to construct a simulation that could be successfully applied to experimental data for data analysis. However, in another aspect, the objective of this theoretical approach was not truly fulfilled. While a complete understanding of the Glauber Model was supposed to be acquired, several questions that developed during this analysis that remain unanswered. The most prominent question that remains unanswered is why the two versions of the Glauber Model differ in their calculations for participant and collision multiplicity. If they differ in these calculations, it is certainly possible that they differ in other calculations as well. In the future, it would be interesting to see whether or not considering nucleon-nucleon potential energy would change the Monte Carlo Glauber's calculations by preventing nucleons from laying on top of each other. It would also be interesting to see if the divergence of the calculations from these two versions decreased as atomic number decreased. Another question that still remains is why the inelastic cross-section remained constant throughout several nucleon-nucleon interactions. If a nucleon-nucleon interaction truly occurred, why wasn't energy lost from one nucleon as it interacted with the other nucleon? Wouldn't a nucleon eventually lose enough energy after a certain number of collisions to no longer be able to interact with other nucleons? It would also be interesting to attempt to track the number of collisions each nucleon had and change the amount of remaining energy they had after each collision.

While these questions may remain unanswered until the distant future, this theory did produce a simulation that was applicable to experimental data, which is available for future data analysis.

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