# Practice Midterm
# Physics 104A

(actual test from November 2005: median was 50, range 15 to 93)

Do all questions. You can often do later parts of a question even if you didn't solve the first or second part. The test has 100 points total, so pace yourself appropriately.

**1.** (10 points)
Evaluate $\int_0^2 \delta((2x-1)\cos x)(x^2-1)dx$. (Since you don't have calculators, you answer won't simplify to anything very nice.)

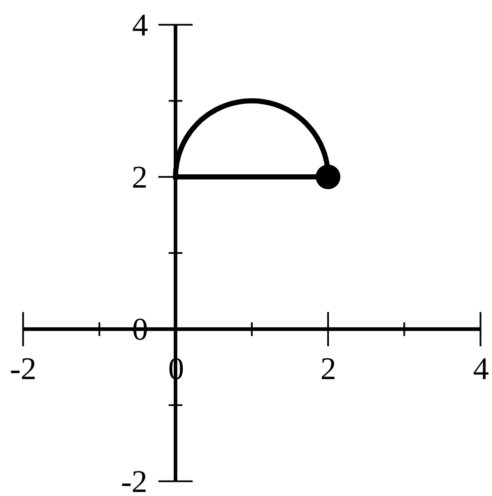
**2.** (15 points)

**a)** The points (0,1,0), (1,1,1), and (1,2,3) determine a plane. Name a point *not* on the plane.

**b)** Find two vectors in the plane of part a), orthogonal to each other.

**c)** Find a vector perpendicular to the plane of part a).

**3.** (10 points)




Sketch the image of the above semicircle (and the marker dot at one vertex) under the transformation $A = \begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix}$.

**4.** (15 points)
Consider the series $\sum_{n=0}^{\infty} a_n x^{2n} = 1 + x^2 + \frac{3}{2}x^4 + \frac{5}{2}x^6 + \frac{35}{8}x^8 + \frac{63}{8}x^{10} + \frac{231}{16}x^{12} + \frac{429}{16}x^{14} + ....$
(The pattern is $a_n = \frac{2n-1}{n}a_{n-1}$, but you don't need to know this to do the problem.)

**a)** Calculate $(\sum_{n=0}^{\infty} a_n x^{2n})^2$ up to the $x^8$ term.

**b)** You should see a pattern in your answer from part a). Use it to evaluate the original series.

**c)** Would the original series converge for $x = \frac{2}{3} + \frac{1}{2}i$? Explain your answer.

**5.** (20 points)

Let

$$f(x) = \begin{cases} 0 & -1 \le x \le 0 \\ \sin \pi x & 0 < x \le 1 \end{cases}$$

**a)** Calculate the exponential Fourier expansion of $f$ on [-1,1] and sketch several periods of the corresponding function. (Be careful not to get bogged down in the calculation.)

**b)** Write down an expression for the sine-only Fourier expansion of $f$ on [-1,1] and a formula for the coefficients. (You do *not* need to evaluate them.) Sketch several periods of the corresponding function.

**c)** Name a function even (symmetric) about $x = 0$ and orthogonal to $f(x)$.

**6.** (10 points)

Let $Z$ and $C$ be operators that act on the set of continuous complex functions $f(z)$, where $z$ is a complex number. Here $Z$ is multiplication by $z$, and $C$ is complex conjugation. Find an operator $M$ such that the commutator $[Z, C]$ equals $CM$.

**7.** (20 points)

For all parts of this problem, assume that the matrix $A$ has a set of orthogonal eigenvectors.

**a)** You can find a matrix $S$ that diagonalizes $A$ and has one of the following properties; which one? (antisymmetric, symmetric, real, imaginary, orthogonal, anti-Hermitian, Hermitian, unitary)

**b)** Not every matrix that diagonalizes $A$ has the property of part a). Explain two ways that $S$ might *not* have that property. (One applies to any matrix $A$ with orthogonal eigenvectors, the other only to certain $A$.)

**c)** Consider $(S^{-1}AS)^\dagger$. You can manipulate this expression in two ways: by simplifying the Hermitian conjugate of a product, and by considering what Hermitian conjugation does to a diagonal matrix. Show that if $A$ has all real eigenvalues, it must be Hermitian.

**d)** If the eigenvalues of $A$ are *not* all real, what condition on $A$ and $S$ does the argument of part c) reach? (It is not especially pretty.)