Midterm, Physics 104A

November 5, 2009

Do all questions. You can often do later parts of a question even if you didn't solve the first or second part. To get full credit, **you must explain your answers**.

1. (15 points)

a) Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$.

b) If complex values of x are allowed, on what interval along the *imaginary* axis does the power series in part a) converge?

c) Does $\sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{n!}$ converge? (This part is completely independent from the previous two.)

2. (15 points) Evaluate $\int_{-\infty}^{\infty} \delta(3x^3 + 2x^2 - x) \cos \frac{\pi}{2} x dx$.

3. (15 points) The functions $f(x) = x^2 + x + 1$ and $g(x) = x - 1$ are orthogonal on $[0, L]$. Find L.

4. (20 points) $f(x)$ is defined on the interval $[0, \sqrt{2}]$ by

$$f(x) = \begin{cases} e^x & 0 < x < 1 \\ e & 1 < x < \sqrt{2} \end{cases}$$

a) What arguments of sine and cosine functions appear in a mixed sine/cosine Fourier expansion?

b) Write down the sine-cosine expansion formula (so I can see how your coefficients are defined).

c) In your expression from b), calculate the coefficient for the fifth sine term.

5. (20 points) Eigenvectors of $\begin{pmatrix} -9 & 32 & -44 & 4 \\ -24 & 43 & -40 & 8 \\ 0 & -4 & 19 & 4 \\ -24 & 20 & -32 & 31 \end{pmatrix}$ include $\begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ -2 \\ -2 \\ 0 \end{pmatrix}$.

a) Find the eigenvalues of the matrix.

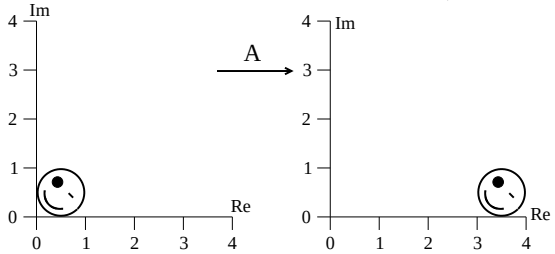
b) Find a general expression for *all* eigenvectors of the matrix.

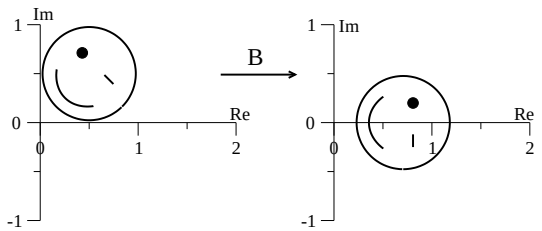
c) How many distinct eigenvectors (not just differing by a scale factor) are orthogonal to $\begin{pmatrix} 1 \\ -2 \\ -2 \\ 0 \end{pmatrix}$?

d) Find all the orthogonal eigenvectors from part c).

6. (15 points)

a) What simple operators do the maps shown in A and B represent, in the complex plane? (Think basic arithmetic operations.)





b) Calculate how the commutator of the two operators from part a), $C \equiv [AB - BA]$, acts on an arbitrary complex point $z = x + iy$.

Extra Credit. (5 points) What does the commutator operator C that you calculated in 6b) mean? (How does it act on the complex plane? How could you represent it as you did in 6a) for A and B?)