Problem Set 7
Physics 104A

Due Tuesday November 10, 2009 at start of class
Late HW accepted for half-credit until class November 12
Counts as a half-problem set only for grading purposes

Primary topic: calculating Fourier series expansions

Do Boas, 7.6.5 and 7.13.9 (the latter is 7.12.9 in the second edition); plus the following.

1. a) Expand $g(x) = \cos x + \sqrt{7} \sin 5x$ in a Fourier series of complex exponentials on the interval $[0, 2\pi]$, and also in a mixed sine-cosine series on the same interval.

b) Expand $f(x) = e^{ix/3}$ in a Fourier series (of exponentials) on $[0, 2\pi]$.

c) Use your answer from b) to expand $f(x) = \sin \frac{x}{3}$ in terms of $\sin nx$ and $\cos nx$, for integer n. Why is the expansion not simply $f(x) = \sin \frac{x}{3}$ itself, since the function is a sinusoidal?

2. a) Expand the Dirac delta function $\delta(x)$ in an exponential Fourier series on $[-\pi, \pi]$.

b) Show that $\delta(\phi_1 - \phi_2) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{im(\phi_1 - \phi_2)}$ is a Dirac delta function by showing that it satisfies the definition of a Dirac delta function, $\int_{-\pi}^{\pi} f(\phi_1) \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{im(\phi_1 - \phi_2)} d\phi_1 = f(\phi_2)$. (Hint: represent $f(\phi_1)$ by an exponential Fourier series.)

And a typical test problem that needn't be handed in:

T1. (18 minutes)
Let $f(x)$ be a step function: $f(x) = 0$ for $0 < x < 1$ and $f(x) = 1$ for $1 < x < 2$.

a) Expand f in a mixed sine/cosine Fourier expansion on [0,2].

b) In the expansion of part a), many of the coefficients vanish. Explain why you should expect them to vanish even without doing any integrals.

c) Does the Fourier expansion converge absolutely? Does it converge uniformly?

d) One could instead expand f in a sine-only Fourier expansion on [0,2] . Sketch the function the sine-only expansion would converge to on [-4,4].