Problem Set 4
Physics 104A

Due Tuesday October 20, 2009 at start of class
Late homework may be turned in by class Thursday October 22 for half credit

Primary topics: power series; vector spaces
Do Boas 3.5.30, 3.5.35, 3.7.7, and 3.7.10, and the following. (The questions are the same in both editions.) As usual, you need not turn in the "T" problems.

1. Write a computer (or calculator) program which calculates the partial sums of an infinite series. For the series $\sum_{n=1}^{\infty} x^n$, graph $s_N = \sum_{n=1}^{N} x^n$ as a function of N, first with $x = 0.1$ and then with $x = 0.9$. Find the smallest N with s_N within 1% of the exact sum, for both $x = 0.1$ and $x = 0.9$. Also graph s_N vs N for the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

2. For each of the five series in problem 4, HW Set 3, state whether the series converges uniformly on its entire range of convergence.

3. For $\mathbf{v} = 2\hat{\mathbf{x}} + 3\hat{\mathbf{y}}$, $\mathbf{w} = -\hat{\mathbf{x}} + \hat{\mathbf{y}}$ calculate the following:

a) $\mathbf{v} \cdot \mathbf{w}$
b) $\mathbf{v} \times \mathbf{w}$
c) θ, the angle between $\mathbf{v}$ and $\mathbf{w}$
d) $\hat{\mathbf{v}}$, the unit vector in the direction of $\mathbf{v}$

4. When are the scalar products $(a, b, c) \cdot (a, b, c)$ and $(a, b, c) \cdot (b, c, a)$ equal? When one is larger, which one is it? Explain. (No messy calculations necessary!)

5. a) Find an orthonormal basis by the Gram-Schmidt method, starting with the three vectors (1,1,0), (1,1,-1), and (3,0,4).

b) Construct orthonormal functions $g_1(x)$, $g_2(x)$, $g_3(x)$ on [0,1] from $F_1(x) = 1$, $F_2(x) = x$, and $F_3(x) = e^{-x}$. (Please keep that order for the Gram-Schmidt procedure.) What is the "angle" between F_1 and F_2?

c) Write $h(x) = 3e^{-x} - 2$ in terms of your orthonormal set from b).

T1. (4 minutes)
Let $\mathbf{v} = (1, 1/3, 1/9, 1/27, 1/81, ...)$ be an infinite-dimensional vector. What is $\mathbf{v} \cdot \mathbf{v}$?

T2. (8 minutes)

a) Normalize $f(x) = \sqrt{x}$ on [0,2].

b) Find a function orthogonal to $f(x) = \sqrt{x}$ on [0,2]. Your function need not be normalized, but it must be normalizable.