# Problem Set 3
# Physics 104A

Due Tuesday October 13, 2009 at start of class

Late homework may be turned in by class Thursday October 15 for half credit

Primary topic: infinite series

**1.** Rank the following expressions from smallest to largest for large $n$: $10^n$; $n!$; $53{,}000{,}000 \times 5^n$; $n^{10}$; $\sqrt{n!/2}$.

**2.** Do the following series converge or diverge? Give a one-sentence explanation for each. Most can be done by comparison with $A\sum 1/n$, $A\sum 1/n^2$, or $A\sum 1/2^n$. The remainder can be done with the integral test. Don't bother with any fancy convergence tests!

a) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

b) $\sum_{n=1}^{\infty} \frac{n^n}{10^n}$

c) $\sum_{n=1}^{\infty} \frac{1}{nn^{1/n}}$

d) $\sum_{n=2}^{\infty} \frac{1}{n(n-1)}$

e) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

f) $\sum_{n=1}^{\infty} \frac{1}{n2^n}$

**3.** Redo problem 2 with each series an alternating series. For example, 3a) becomes $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$. State whether each alternating series converges absolutely, conditionally, or not at all.

**4.** For what range of $x$ are the following series convergent? Are there any values of $x$ for which they converge conditionally? (You should give BRIEF justifications of your answers.)

a) $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n}$

b) $\sum_{n=1}^{\infty} \frac{x^{3n}}{n}$

c) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

d) $\sum_{n=0}^{\infty} \frac{x^n}{(n!)^2}$

e) $\sum_{n=0}^{\infty} \frac{1}{1+x^n}$

**5.** Expand $f(x,y) = \sqrt{1+x+y^2}$ around $x=0, y=0$ to third order in $x$ AND $y$. (That is, up to the $x^3$, $x^2y$, $xy^2$, and $y^3$ terms.) The calculation can be moderately ugly or really horrendous, depending on the order you do things, so do think a bit before plunging ahead!

**6. a)** Expand $\frac{1}{z+i}$ about $z=0$ (to cubic terms in $z$) and about $z=2$ (to cubic terms in $z-2$). What is the radius of convergence in each case? One of the expansions converges at the center point of the other; which one?

**b)** Although the two series you found may appear different, they must coincide at points inside the radius of convergence of both, since they converge to the same function. Verify that the constant terms are indeed the same: in the $z=2$ expansion, multiply out the powers of $z-2$ to find ALL constant terms. The constant terms form an infinite series, and the pattern should become apparent after the first few terms. Sum this infinite series; the answer should equal the constant term in the $z=0$ expansion. You could carry out similar sums for the $z$, $z^2$, $z^3$. Why does this only show that the series coincide within the radius of convergence?

**7.** Start with the harmonic series, $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...$, and remove all terms with at least one "7" in the denominator, such as $\frac{1}{7}$, $\frac{1}{37}$, $\frac{1}{701}$, $\frac{1}{8770}$, etc. Does the remaining series converge?

I would not ask question 6 (too gory) or question 7 (too hard, except perhaps as extra credit) on a test. The others are all plausible though, and here's one more that need not be turned in.

**T1.** (8 minutes)

a) Find the interval of convergence of the Taylor expansion, around $x=0$, of $f(x) = \frac{1}{(x-3)(x^2+4)}$. (Don't worry about whether the expansion converges at the endpoints of the interval.)

b) For what real $x$ does $\sum_{n=1}^{\infty} \frac{x^3}{4(n^2+n)}$ converge?