

Problem Set 2

Physics 104A

Due at start of class on Tuesday October 6, 2009
Homework accepted for late credit until start of class Thursday October 8

Primary topic: Dirac delta function

1. Evaluate the following integrals.

a) $\int_0^\infty f(x)\delta(2x-3)dx$

e) $\int_{-\infty}^\infty e^x\delta(x^3)dx$

b) $\int_0^{2\pi} \frac{\sin z}{\cos^2 2z + \tan^2 z/2} \delta(\pi-z)dz$

f) $\int_0^{2\pi} \sin(\theta)\delta(e^\theta \cos \theta)d\theta$

c) $\int_0^3 f(x)\delta(2x^2-x-1)dx$

g) $\int_{-\infty}^\infty f(x)\delta(2x^2+1)dx$

d) $\int_{-\infty}^\infty \cos x\delta(x^3-\pi^2x)dx$

h) $\int_0^r f(x)\delta(\sqrt{r^2-x^2}-2)dx$

(Take two cases, $r > 2$ and $r < 2$.)

2. Evaluate the following integrals.

a) $\int_{-\infty}^\infty f(x)\delta'(x-1)dx$

b) $\int_{-5\pi/2}^{5\pi/2} e^{-2x}\delta'(\sin x)dx$

c) $\int_{-\infty}^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty dx dy dz f(x,y,z)\delta(\frac{x}{2}+1)\delta(y-3)\delta(\sqrt{6}z+1)$

d) $\int_{-\infty}^\infty \int_{-\infty}^\infty dx dy (x^2+xy+1)\delta(3x-5)\delta(y)$

e) $\int_0^\infty dx \int_0^\infty dy \frac{\delta(x-y)}{(4x^2+5y^2+1)^{3/2}}$

Also integrate the same integrand over the upper half-plane (i.e., $y > 0$, $-\infty < x < \infty$) and over the whole plane.

f) $\int_0^{\pi/4} \int_0^{\pi/4} dx dy \delta(x-2y) \cos \sqrt{2xy}$

Do this one in both orders. (That is, x-integral followed by y-integral and vice versa.) If you don't get the same answer both ways, think carefully about your integration limits.

Once again, almost any of this might show up on a test. (In fact, I have used 1f, 1g, and 2d, allowing about 4 minutes apiece.) Here's one more, which need not be turned in.

T1. (8 minutes) Evaluate the following integrals.

a) $\int_{-\pi/2}^{\pi/2} (\cos \theta)\delta((\theta - \frac{\pi}{4}) \sin \theta)d\theta$

b) $\int_0^\infty \left[\int_0^5 \delta(x-y)dx \right] dy$