Problem Set 1, Physics 104A

Due at start of class Thursday October 1, 2009
No late homework accepted. (Due date already delayed due to walkout.)

Primary topics: complex algebra, Euler's formula
Secondary topic: graphing complex functions and transformations

Do NOT use an integral table, graphing calculator, symbolic computation program, or other such aids. You may use a calculator for real arithmetic, though.

1. a) Write $\frac{3-2i}{2+i}$ in the form $x+iy$. Write the same number in the form $re^{i\theta}$. Graph the number in the complex plane, and identify x, y, r, and θ.

b) Write $(1+i)^{29}$ as $x+iy$ with x and y real.

c) Write $(\frac{1+i}{1-i})^{2718}$ as $x+iy$, with x and y real.

d) Write $\tanh \frac{i\pi}{4}$ as $x+iy$, with x and y real.

e) On a graph of the complex plane, shade the region with $\mathrm{Re}(e^{i\pi/2}z) > 2$.

2. a) Solve for z: $z^5 = 1$. Plot the solutions in the complex plane.

b) Repeat a) for the equation $z^4 = i$.

c) Repeat a) for the equation $z^3 = 2+i$.

d) Repeat a) for the equation $(z-2)^3 = 1$.

e) Repeat a) for the equation $|z-2|^3 = 1$.

3. Do these integrals carefully and be sure you understand them. They are fundamental to Fourier analysis, and we'll refer back to them many, many times.

a) Evaluate $\int_0^{2\pi} e^{-imx}e^{inx}dx$, where m and n are integers. (You will have two cases, depending on the values of m and n.)

b) Evaluate $\int_0^{2\pi} \sin mx \sin nx dx$ and $\int_0^{2\pi} \cos mx \sin nx dx$ by writing cos and sin in terms of complex exponentials and using part a).

4. Consider the complex function $f(z) = e^{-3iz}$.

a) For $z = x$, with x real, graph $\mathrm{Re}(f(z))$ and $\mathrm{Im}(f(z))$ as functions of z.

b) Repeat part a) for $z = ix$, with x real.

c) Sketch the image in the complex plane of the real line under the map f. Be sure you understand how your pictures for parts a) and c) relate to each other.

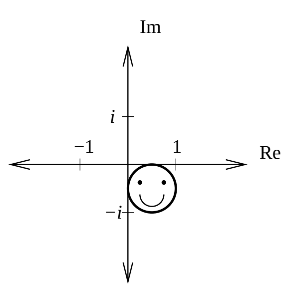
5. Sketch the face's image in the complex plane under the following maps.

a) $f(z) = z + (2-i)$

b) $g(z) = (1+i)z$

c) $f(g(z))$

d) $g(f(z))$



Tests: any of these problems could be used on an exam. I would allow 5 minutes or less for each sub-question.