Answer Set 1, Physics 104A

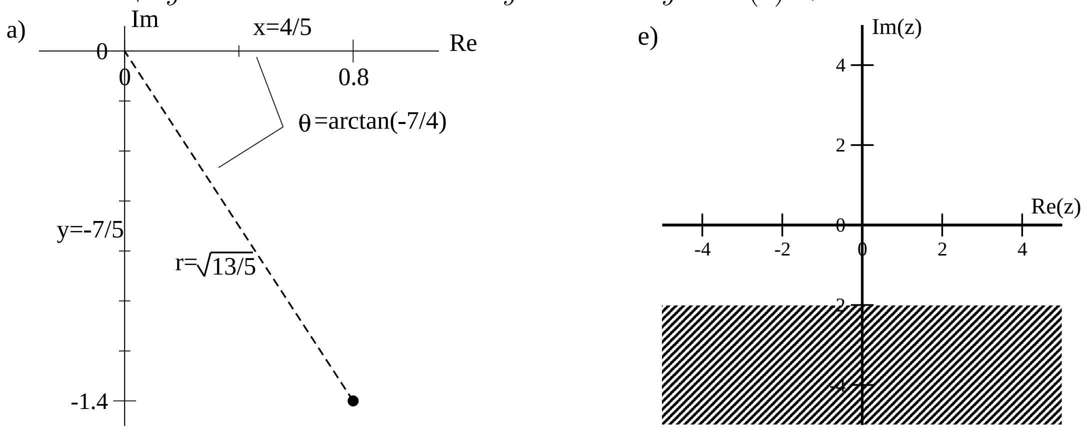
1. a) $\frac{3-2i}{2+i} = 4/5 - 7i/5 = \sqrt{13/5}e^{-1.05i}$, where $-1.05 = \arctan(-7/4)$.

b) Since $(1+i)^2 = 2i$, $(1+i)^{29} = [(1+i)^2]^{14}(1+i) = (2i)^{14}(1+i) = -2^{14} - 2^{14}i$.

c) $\frac{1+i}{1-i} = \frac{(1+i)^2}{2} = i$, so $(\frac{1+i}{1-i})^{2718} = i^2 = -1$. (Use the fact that $i^4 = 1$.)

d) $\tanh\frac{i\pi}{4} = \frac{e^{i\pi/4}-e^{-i\pi/4}}{e^{i\pi/4}+e^{-i\pi/4}} = \frac{e^{i\pi/2}-1}{e^{i\pi/2}+1} = \frac{i-1}{i+1} = \frac{(-1+i)(1-i)}{2} = i$.

e) Write $z = x + iy$. Then $e^{i\pi/2}z = iz = ix - y$ so we need $y = \text{Im}(z) < -2$.



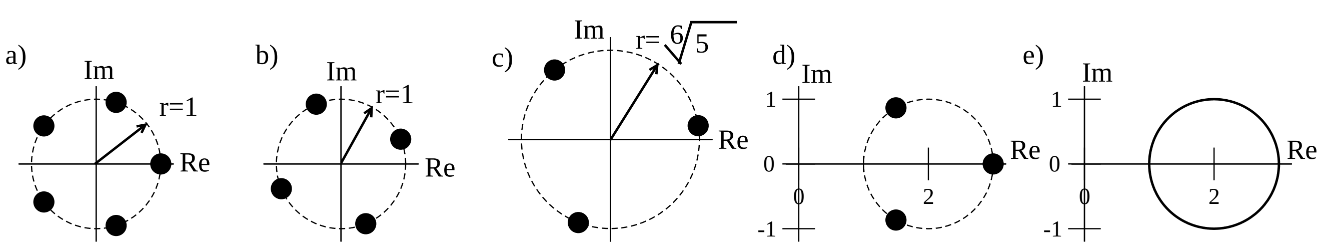
2. a) $z = 1, e^{i2\pi/5}, e^{i4\pi/5}, e^{i6\pi/5}, e^{i8\pi/5}$.

b) $z = e^{i\pi/8}, e^{i5\pi/8}, e^{i9\pi/8}, e^{i13\pi/8}$.

c) $2 + i = \sqrt{5}e^{i\alpha}$, where $\alpha = \arctan(1/2)$. Then $z = 5^{1/6}e^{i\alpha/3}, 5^{1/6}e^{i(\alpha+2\pi)/3}, 5^{1/6}e^{i(\alpha+4\pi)/3}$. In Cartesian coordinates, the first of these is $1.29 + 0.201i$.

d) Write $z - 2 = re^{i\theta}$. Then $(z-2)^3 = (re^{i\theta})^3 = 1 = e^{i0} = e^{i2\pi} = e^{i4\pi}$. This means $r = 1$ and θ is 0, $2\pi/3$, or $4\pi/3$. (Or equivalent angles that differ by 2π from those I listed.) Switching back to Cartesian coordinates gives $z - 2 = 1, -\frac{1}{2} + \frac{\sqrt{3}}{2}$, or $-\frac{1}{2} - \frac{\sqrt{3}}{2}$. Now just add 2 to get solutions for z: 3, $\frac{3}{2} + \frac{\sqrt{3}}{2}$, and $\frac{3}{2} - \frac{\sqrt{3}}{2}$.

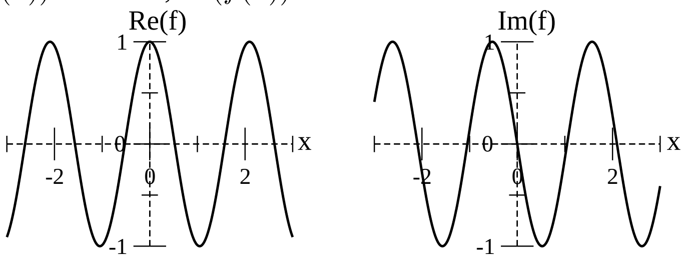
e) This time the solutions are ALL z where $z - 2$ has length 1, which means a circle of radius 1 centered at 2. Note that the three solutions to part d) lie on exactly this circle.



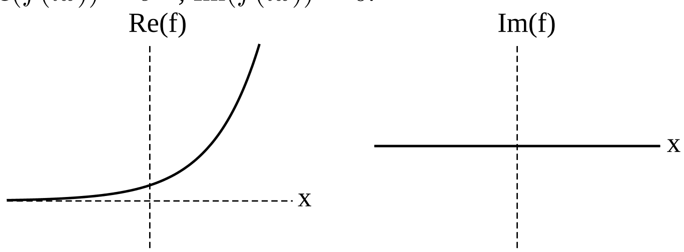
3. a) If $n \neq m$, $\int_0^{2\pi} e^{i(n-m)x}dx = \frac{1}{i(n-m)}e^{i(n-m)}|_0^{2\pi} = 0$. If $n = m$, $\int_0^{2\pi} e^0 dx = \int_0^{2\pi} dx = 2\pi$.

b) I'll assume $m, n \geq 0$. (We can then use $\cos mx = \cos -mx$ and $\sin nx = -\sin -nx$ to do other cases.) $\int_0^{2\pi} \cos mx \sin nx dx = \int_0^{2\pi} \frac{e^{imx}+e^{-imx}}{2}\frac{e^{inx}-e^{-inx}}{2i}dx = \frac{1}{4i}\int_0^{2\pi}(e^{i(m+n)x} - e^{i(m-n)x} + e^{i(n-m)x} - e^{i(-m-n)x})dx$. Each of the four terms integrates to zero unless $m = n$. (For example, $\int_0^{2\pi} e^{i(m+n)x}dx = \frac{1}{i(m+n)}e^{i(m+n)x}|_0^{2\pi} = \frac{1}{i(m+n)}(1-1) = 0$.) If $m = n$ the middle two terms become $\frac{1}{4i}\int_0^{2\pi}(-1+1)dx$, so they add to zero. The integrals of the first and last terms still vanish. $\int_0^{2\pi} \sin mx \sin nx dx = \int_0^{2\pi} \frac{e^{imx}-e^{-imx}}{2i}\frac{e^{inx}-e^{-inx}}{2i}dx$. Same four exponents, so each integral vanishes if $n \neq m$. But this time, if $n = m$, the middle terms have the same sign and the integral is $2\pi(-1-1)/(2i)^2 = \pi$.

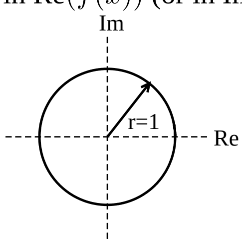
4. a) $\text{Re}(f(x)) = \cos 3x$; $\text{Im}(f(x)) = -\sin 3x$.



b) $\text{Re}(f(ix)) = e^{3x}$; $\text{Im}(f(ix)) = 0$.



c) The real line gets mapped around the unit circle, over and over and over again. One oscillation period in $\text{Re}(f(x))$ (or in $\text{Im}(f(x))$) in part a) corresponds to cycling once around the circle.



5. (Note that c and d are different!)

To do these without a calculator, pick a few "special" points to calculate, and then fill in the rest smoothly. For example, take the top and bottom points in the middle of the original face, and the left and right points halfway up the face.